Rate-and-State Model of Induced Seismicity

Sergey Turuntaev, Alesya Kamay
Institute of Geosphere Dynamics of Russian Academy of Sciences (IDG RAS), Moscow Institute of Physics and Technology, All-Russian Research Institute of Automatics, Moscow

Introduction

In 60s, Brace and Byerlee (1966) proposed to consider unstable frictional sliding along tectonic faults as a model of earthquakes. Peculiarities of the friction force dependence on the duration of the contact stationary state and on the speed of the fault sliding was examined by Dieterich (1992) and other researches. Gu et al. (1984) experimentally studied various modes of the frictional movements and determined empirical constants which are used in many modern variants of the rate-and-state equation.

The origin of the unstable sliding and its dynamics were studied by Ohnaka et al. (1986). The work was focused on the study of mechanism of the transition to instability.

The rate-and-state equation was considered by Hobbs (1990) by means of nonlinear dynamics methods. Change of friction was studied as a function of displacement and velocity at different stiffness coefficients in the rate-and-state equation. The similar approach was implemented by Erickson et al. (2008), they examined an appearance of chaotic solutions in the one-parameter velocity-dependent friction equation.

Here we consider two-parameter type of the friction law and vary the critical shear stress in the rate-and-state equation in suggestion that this is the value varied by human impact (by mining, fluid injection and production, hydraulic fracturing and so on). The obtained solutions of the rate-and-state equation are analyzed by means of Grassberger-Procaccia method (1983).

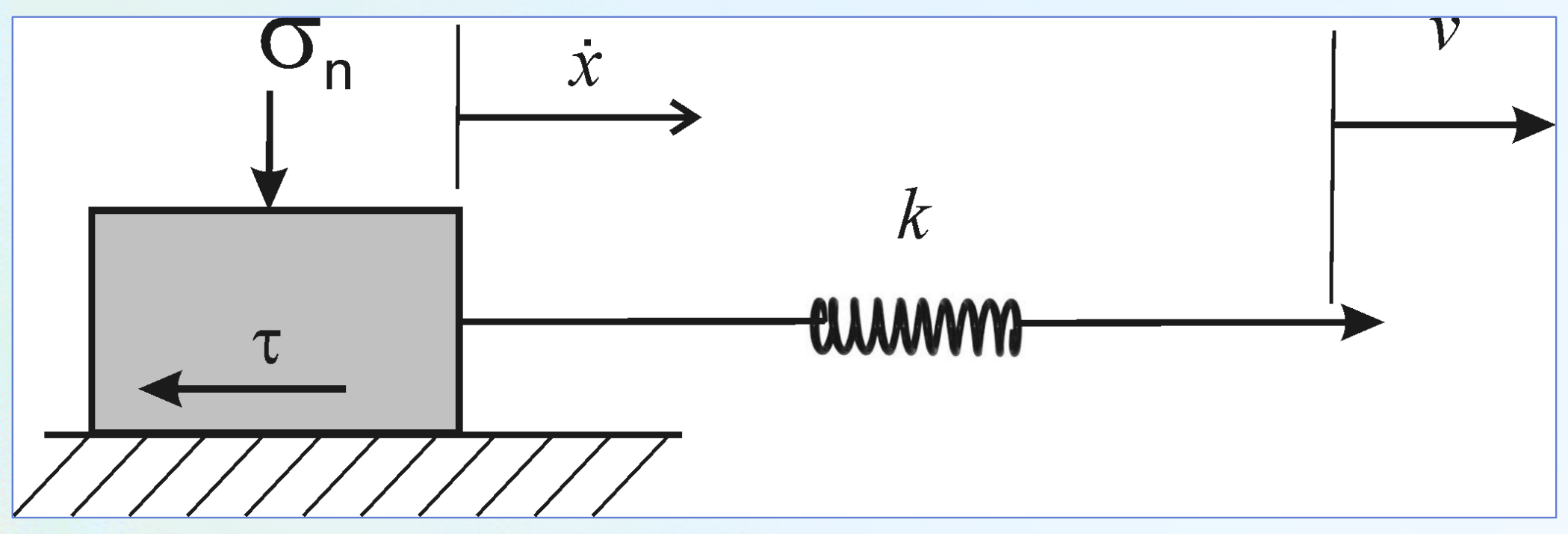


Fig.1. Spring-slider system.

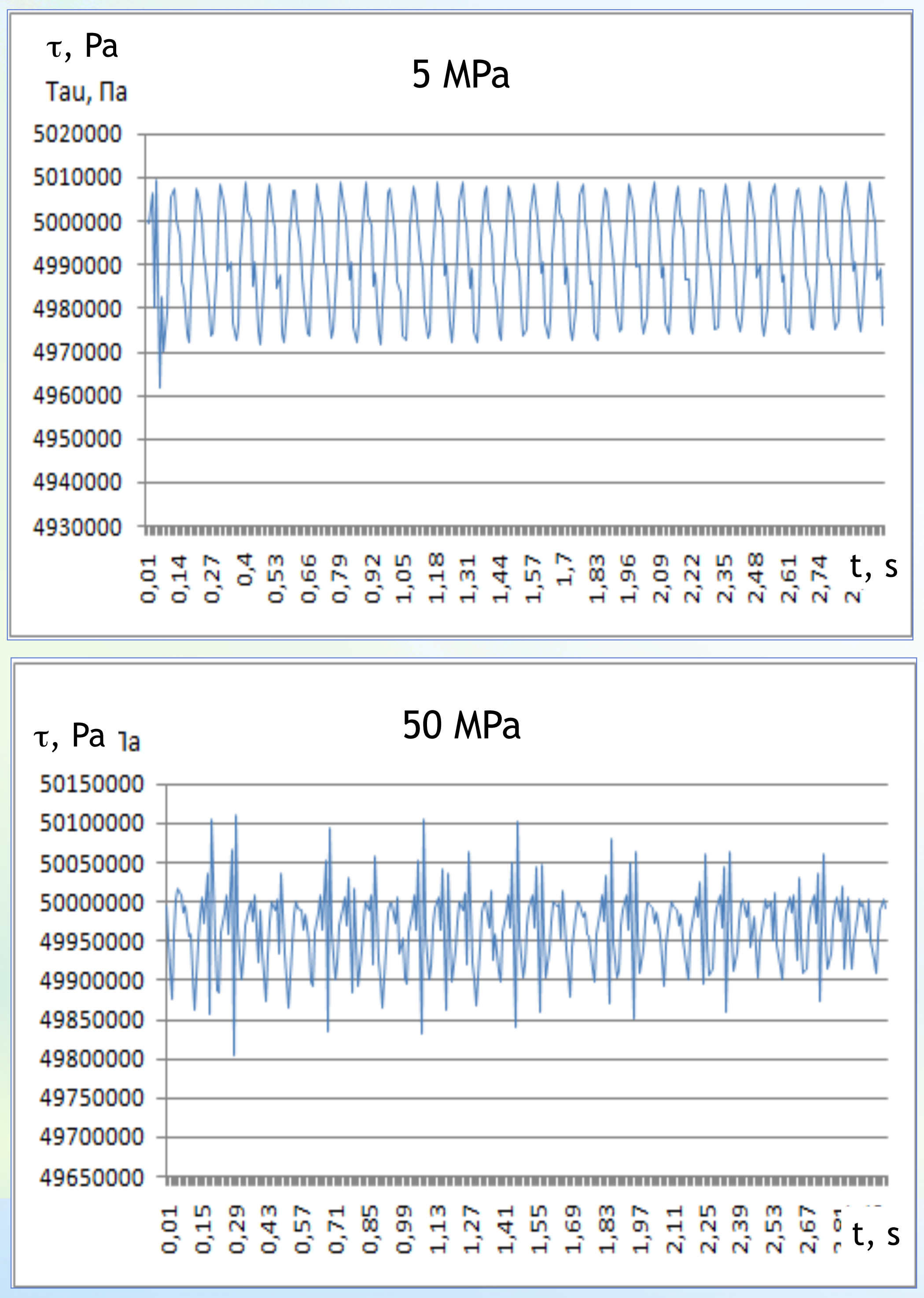


Fig.2. Variations of the shear stress with time at the critical stresses equal to 5 MPa and 50 MPa

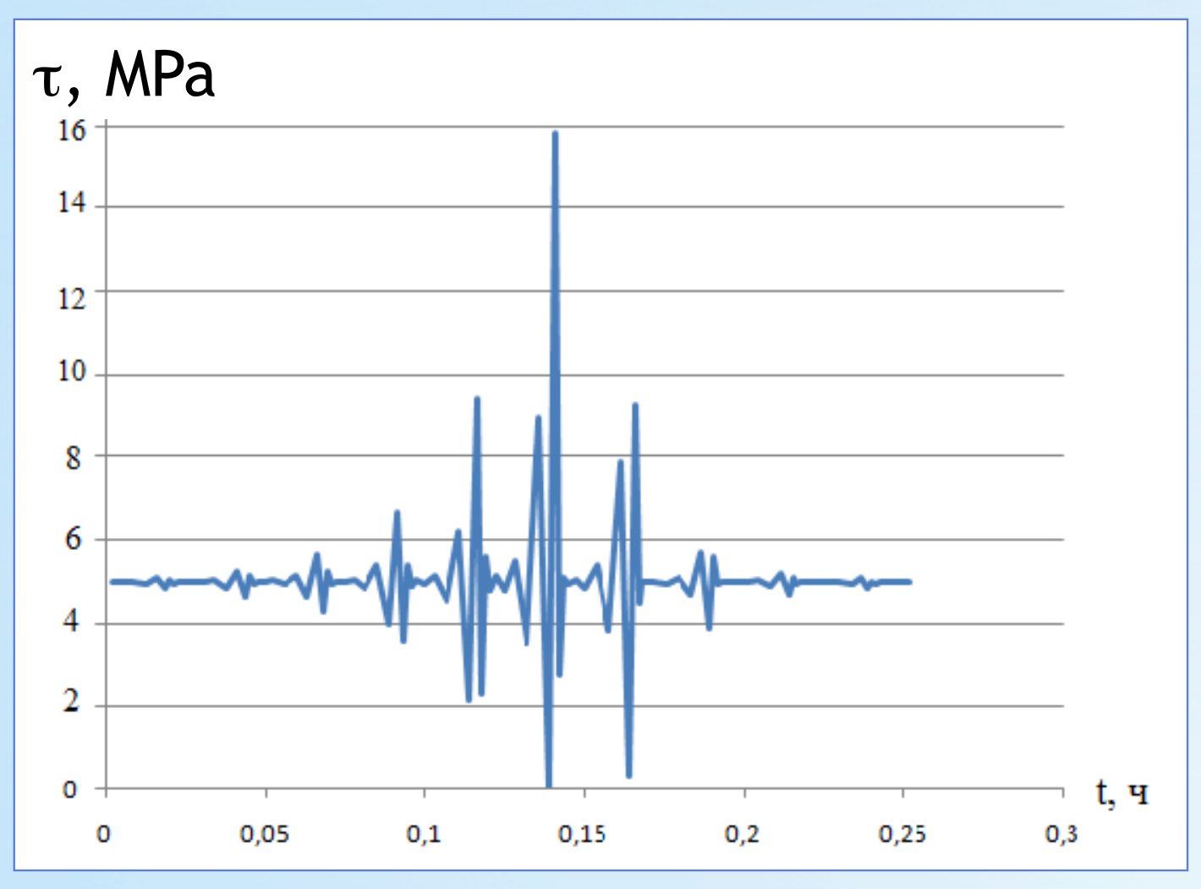
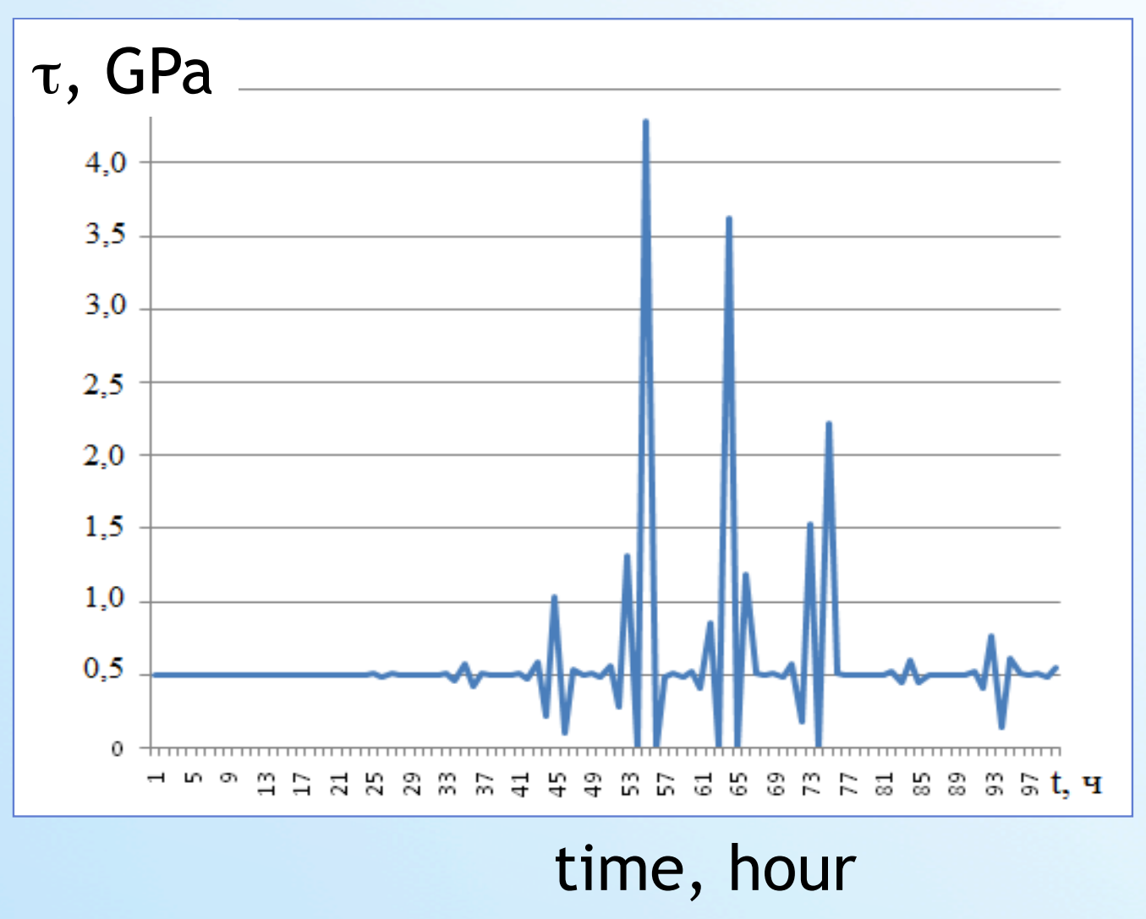


Fig.3. Diminishing of critical stress due to fluid injection results in:

1. Diminishing of time of an earthquake preparation
2. Diminishing of released stress amplitude
3. Change of foreshock-aftershock sequence



The model description

Following the works by Hobbs (1990), Perfettini, Campillo (2003), we suggest that the seismic processes can be described by slider-spring model (Fig.1) with motion equation

$$m\ddot{x} = k(vt - x) - \tau s$$

where friction dependence on the sliding rate can be described by the two-parametric friction law:

$$\tau = \tau^* + A\, ln(\upsilon/\upsilon^*) + \theta_1 + \theta_2$$

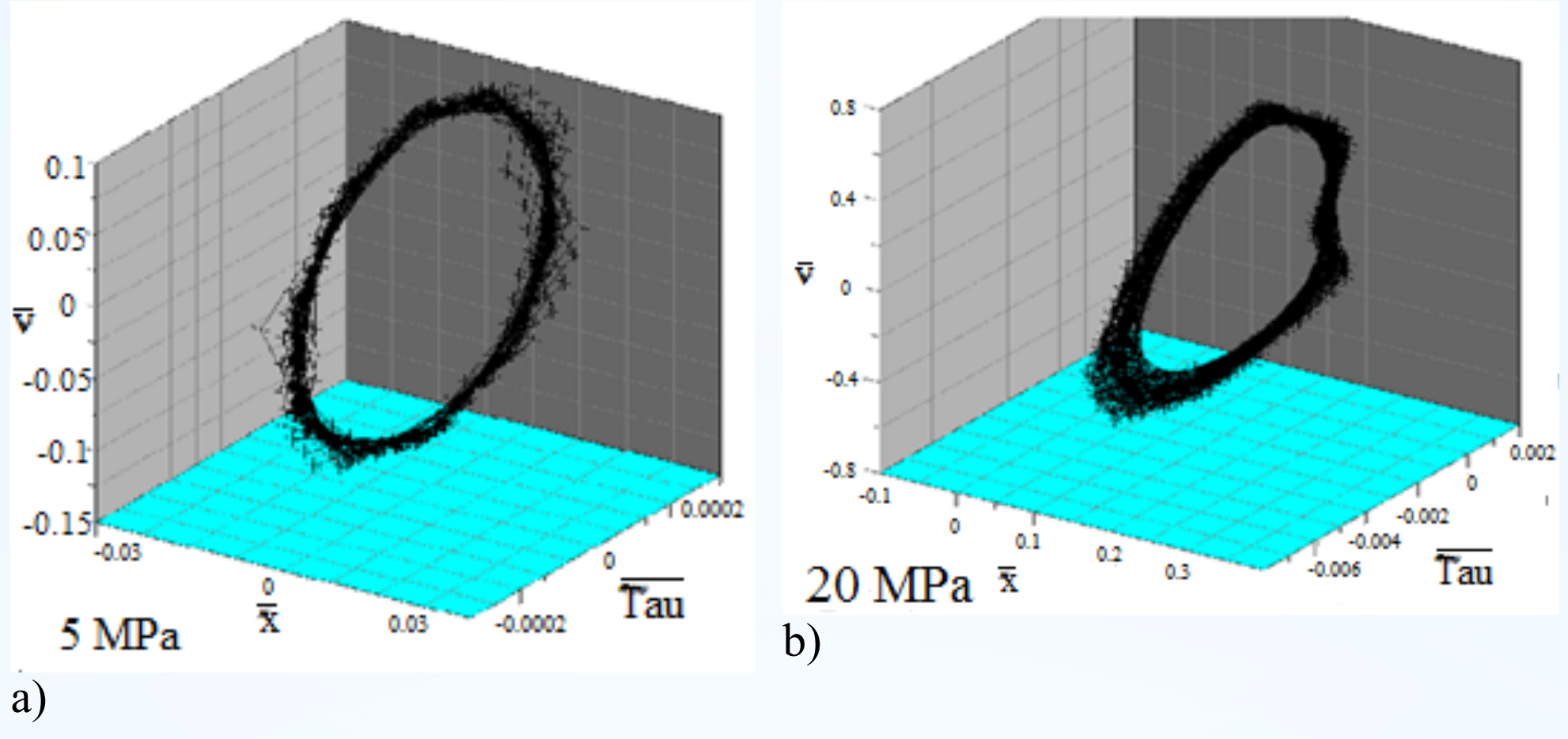
where v^* - parameter, τ^* - critical stress, which can be changed by fluid injection and can be written as

$$\tau^* = C + \mu(\sigma - p)$$

where C - cohesion coefficient, μ - coefficient of friction, p - pore pressure, σ - normal stress; θ_i - state variable, which characterizes the state of the sliding surfaces, and which evolution over time is determined by the equation:

$$\dot{\theta}_i = -\frac{v}{L_i}\left[\theta_i + B_i ln(v/v*)\right]$$

here L_i - characteristic dimensions of the roughness of the sliding surfaces, $i=1,2$. Values of the constants υ^*, A, B_i, τ^*, L_i were taken from experiments of Gu et al (1984).



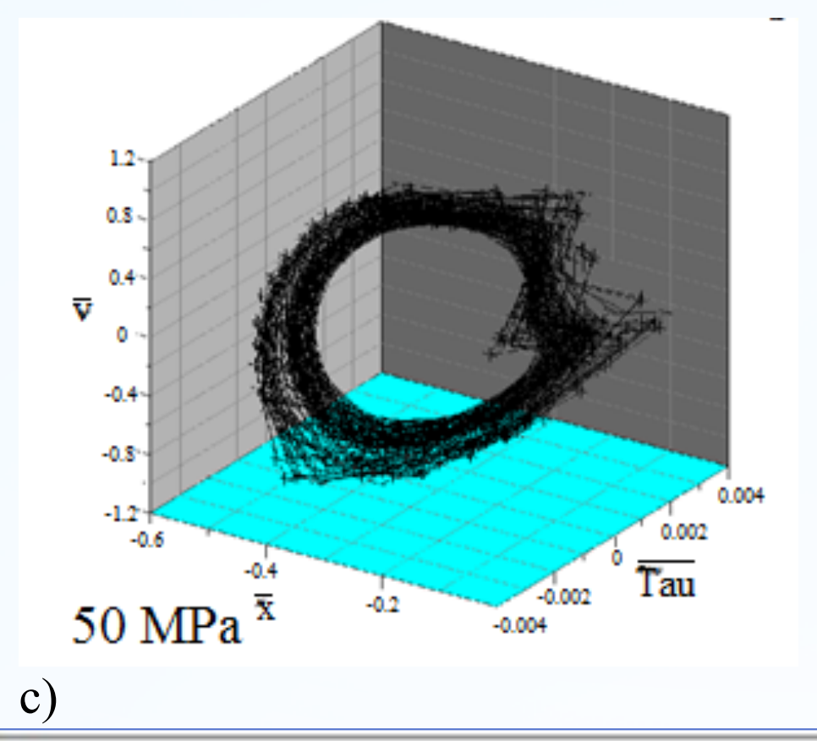


Fig. 4. Phase trajectories of a fault sliding in *x-v-t* coordinates for values of critical shear stresses 5 MPa (a), 20 MPa (b) and 50 MPa (c).

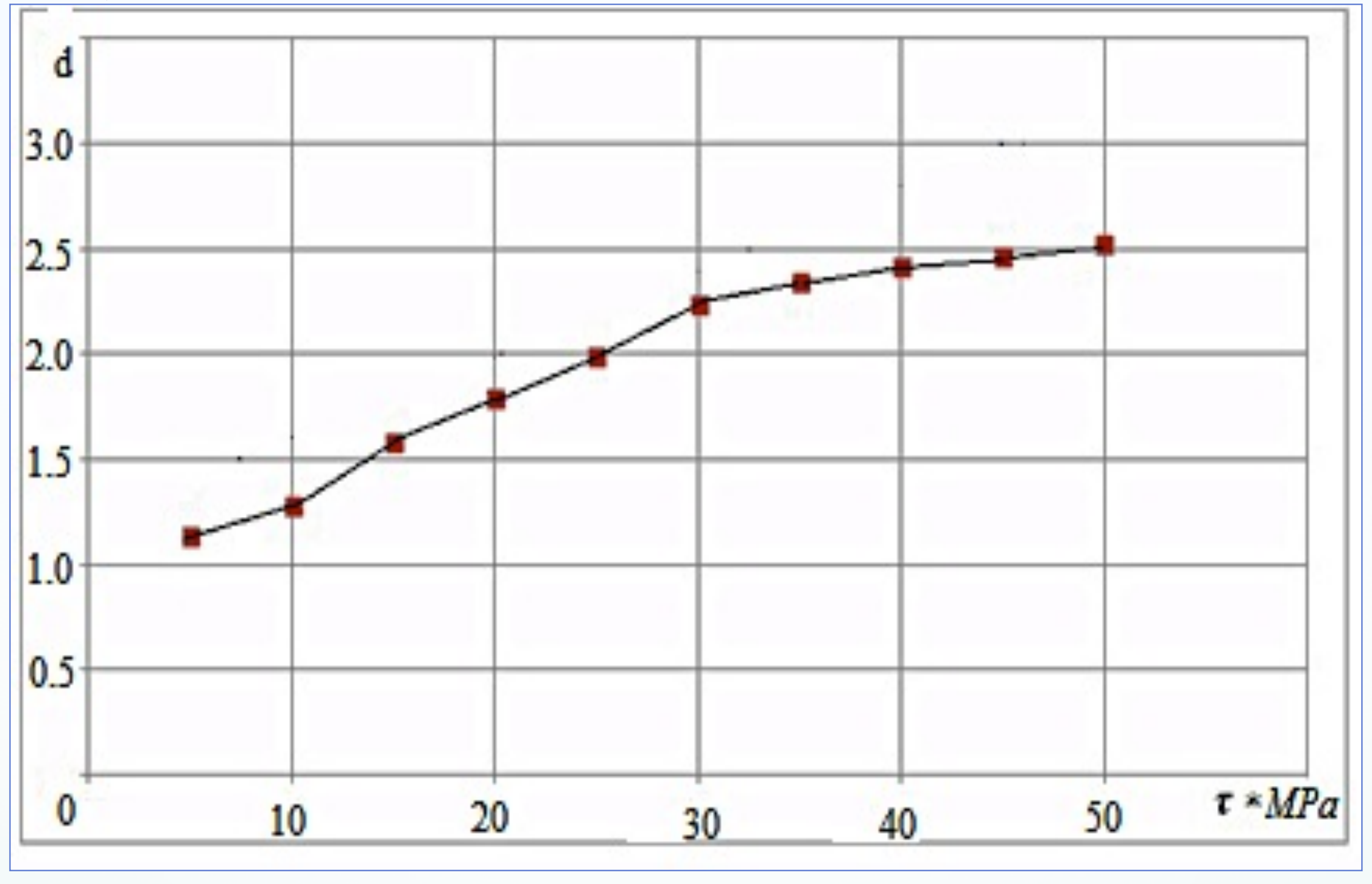
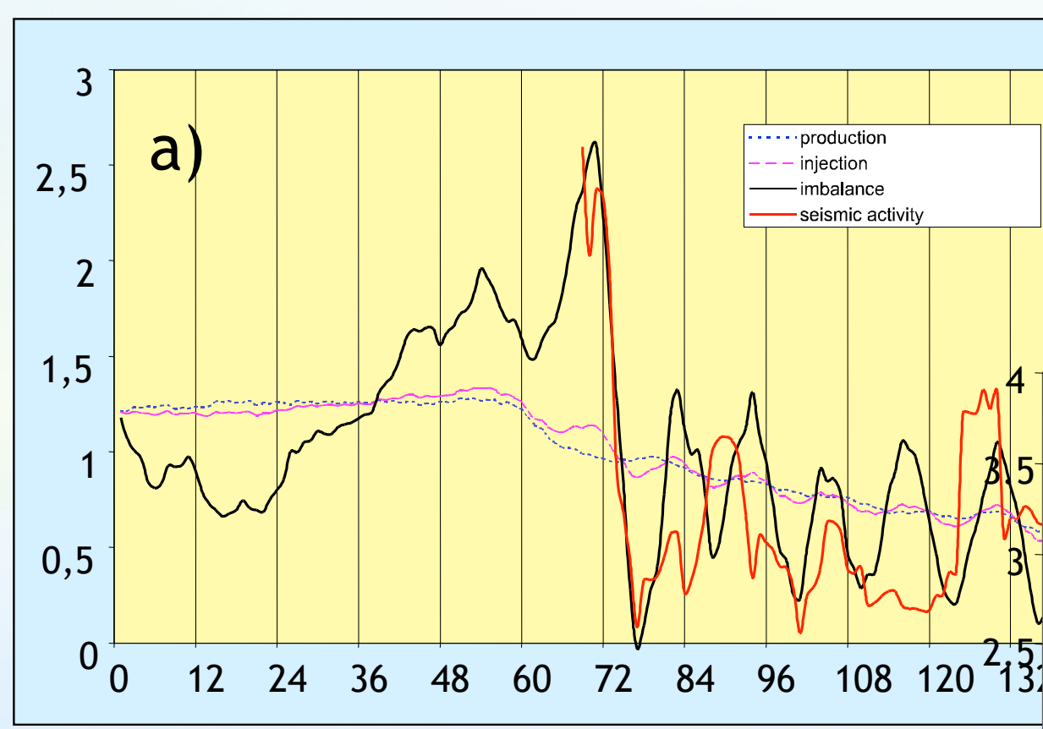


Fig.5. Dependence of correlation dimensionality on critical stresses.



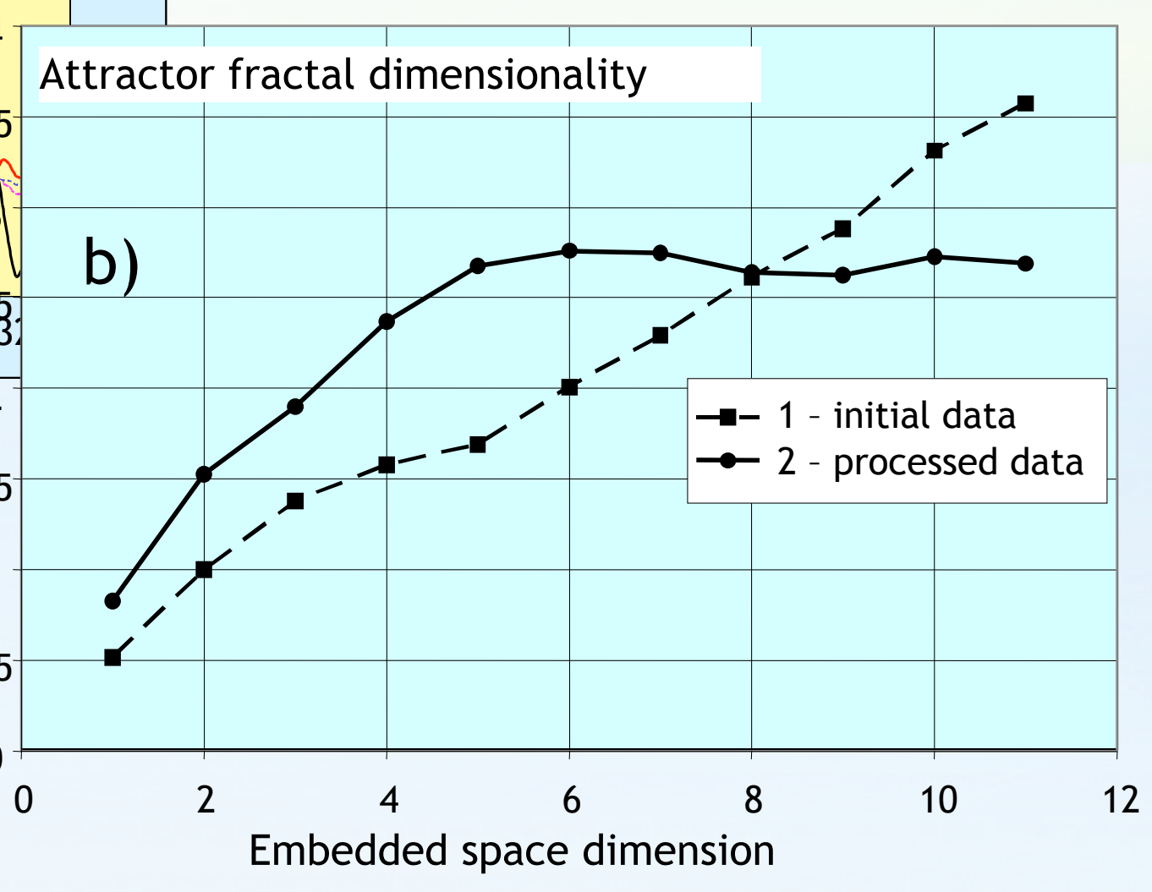
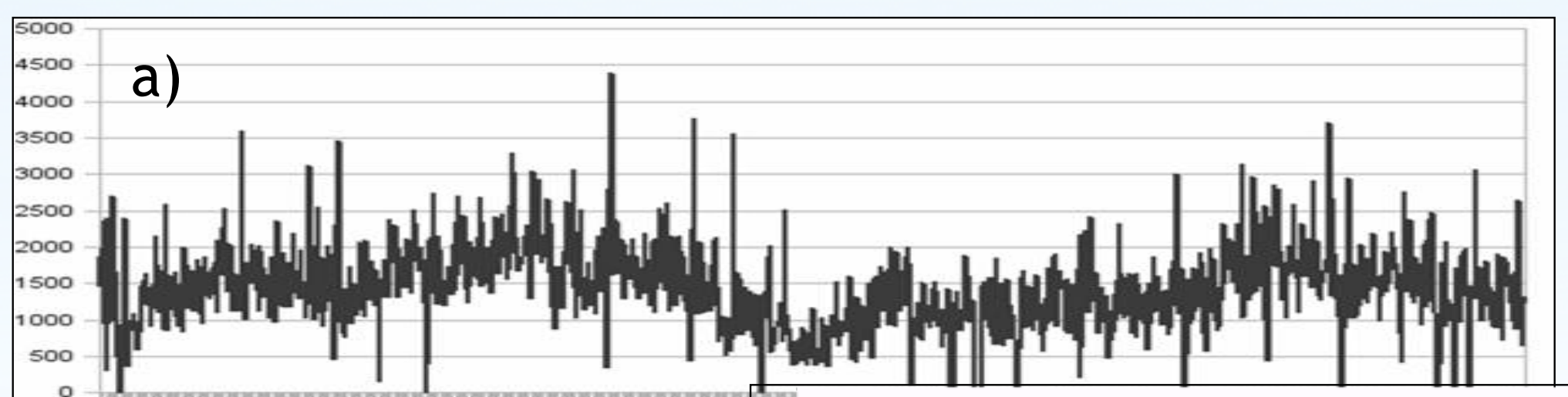


Fig.6. Romashkino oil-field, Russia: a) variations of seismic activity, injection and production volumes and imbalance; b) correlation dimensionality vs. embedding dimensionality



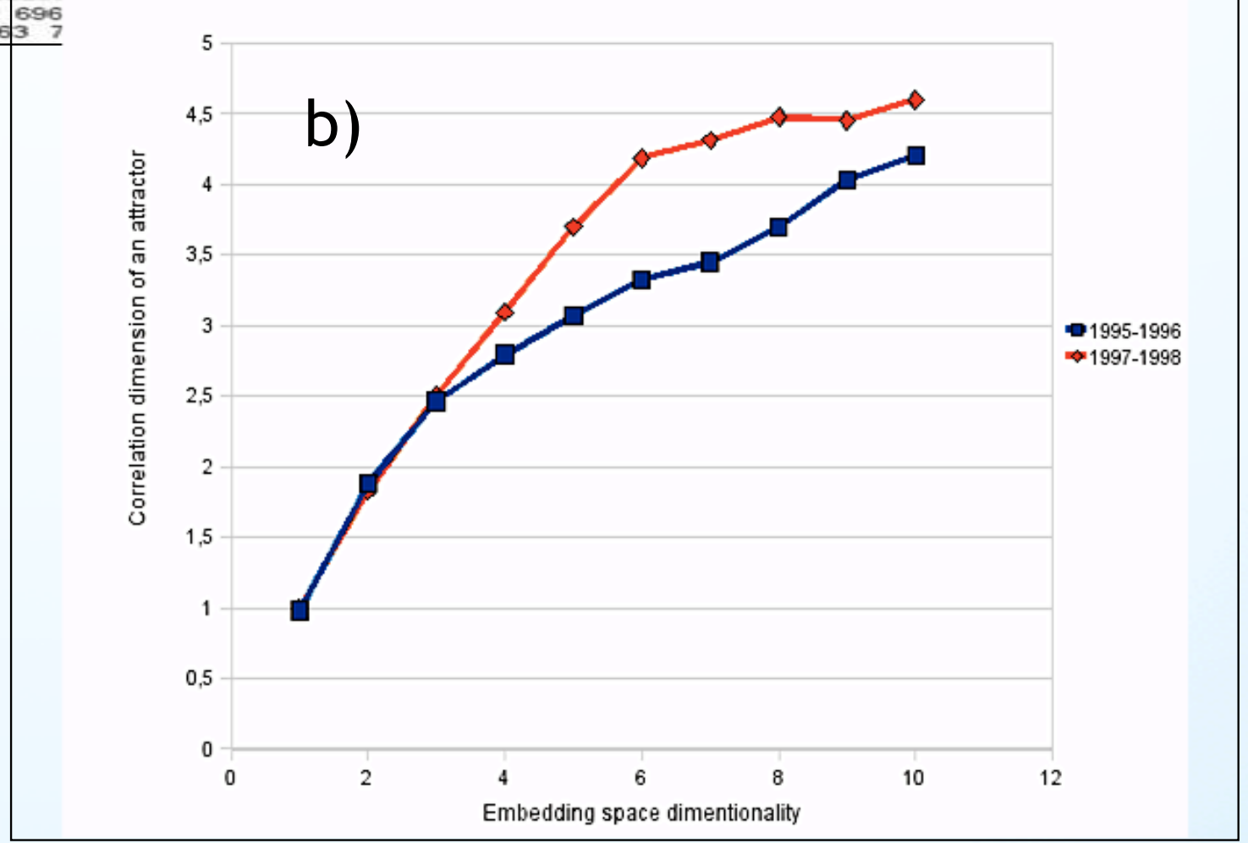


Fig.7. The Geysers: a) seismic activity variations in 1995-1998; b) correlation dimensionality vs. embedding dimensionality

Results

Numerical simulation of the block motions was carried out under the critical stress τ^* varied from 5 MPa to 50 MPa with increments 5 MPa. For each value of τ^*, time series of the block displacements, its velocity and shear stress at the block base were calculated. Complexity of the obtained time series were analyzed using algorithm for estimating the correlation dimension, based on the calculation of the correlation integral by Grassberger and Procaccia method [5].

The graphs of the displacements and the shear stresses for two values of the critical stress τ^* (5 Mpa and 50 Mpa) are shown in Fig.2 and Fig.3 (long time period). Results of the numerical calculations for several values of the critical stresses are shown in Fig.4 as phase trajectories in *x-v-t* coordinates. The values are normalized to the characteristic size L_1, v^* и τ^*=*5 МПа*.

An estimation of the obtained attractor dimensions by Grassberger-Procaccia method showed, that an increase of the critical stresses results in increase of the attractor correlation dimensionality: τ^*=5MPa D^*=1.4; τ^*=15MPa D^*=1.6; τ^* = 30 MPa D^*=2.2; τ^*=45MPa D^*=2.5. (Fig.5).

The same procedure of analysis was applied to data on seismicity in The Geysers area and Romashkino oil field (Fig.6-7). It was found, that in both cases the decrease in correlation and embedding space dimensions are observed after increase of injection volumes (in The Geysers case D^*=4.3-4.5) and beginning of the oil field intensive flooding (in Romashkino case D^*=2.7).

Discussion and conclusions

Numerical analysis of the rate-and-state equation with the two-parameter friction law showed significant changes in the stick-slip motion when the critical shear stress varied.

Evaluation of the correlation dimension and the embedding space dimension by Grassberger-Procaccia method has shown that an increase of critical stress from 5 MPa to 50 MPa resulted in increase of correlation dimension and embedded space dimension from 1.1 to 2.5 and from 3 to 5, respectively.

In the range of the critical stress from 5 MPa to 30 MPa the correlation dimension increases linearly with critical stress increase; at higher values of the critical stress there is a tendency of saturation of the correlation dimension dependence on the critical stress.

Values of dimensions obtained in the numerical modeling may differ from the values, which were obtained in the analysis of real seismicity (for example, in the area of the Bishkek geodynamic test site, see Turuntaev et al. (2012)). We can assume that this difference is caused by significantly higher complexity of real seismic processes in comparison with the model. This distinguish can be explained by taking into account the presence of the seismic events, which are not related with human influence and which can be considered as a stochastic background. An addition of random component with signal/noise ratio 2 to the model data resulted in increase of the model correlation dimensionality to 4-5, which is in good correspondence with induced seismicity data.

It can be concluded that an increase of the critical stresses in the rate-and-state equation results in increase of the attractor correlation dimensionality. It was found, that if the critical stress continue to increase, the correlation dimension would stop to increase. The existence of the stable states in the equation solution allows us to specify the problem of seismic activity forecast and of seismic regime control technologies.

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