



# Partitioning of seismic and aseismic strain

Ian Main

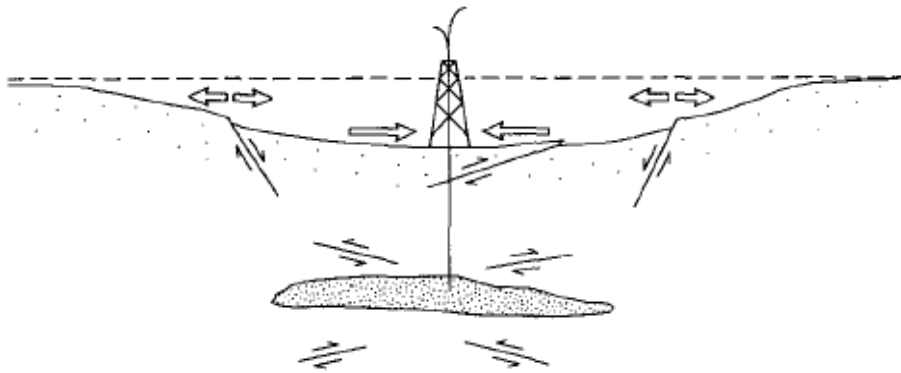
John Greenhough, Ferenc Kun, Imre Varga,  
Sabine Lennartz-Sassinek, Michael Zaiser



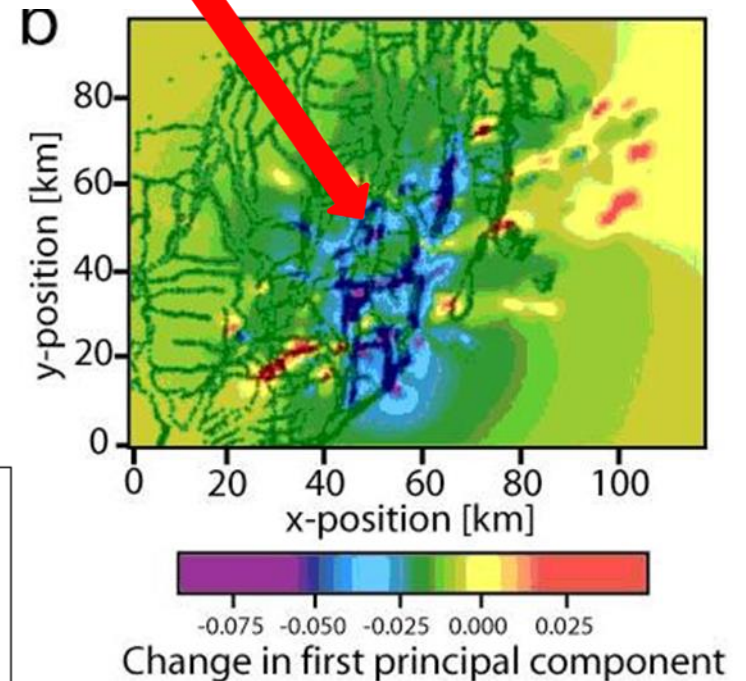
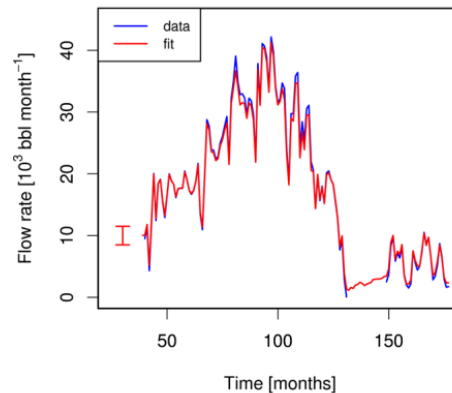
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# Partition of Scalar Strain

Total Strain = (Poro-elastic + Seismic + Silent permanent ) strain



Segall, 1989



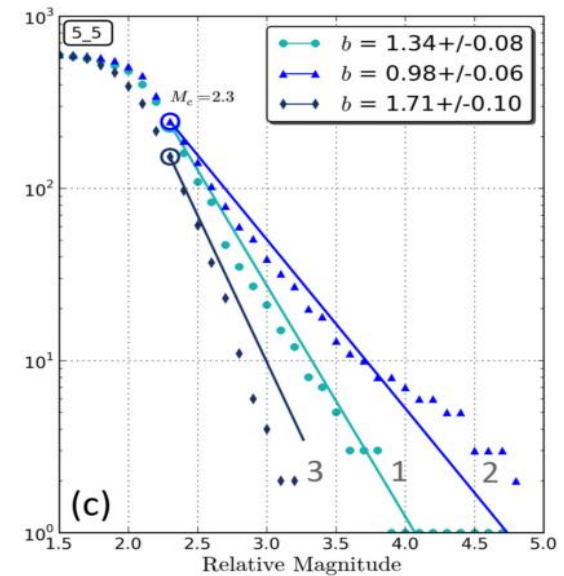
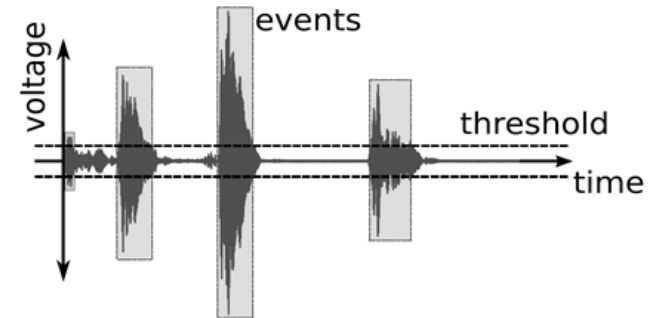
Main et al., GRL, 2006

# Learning from the lab

## Acoustic Emissions (AE)

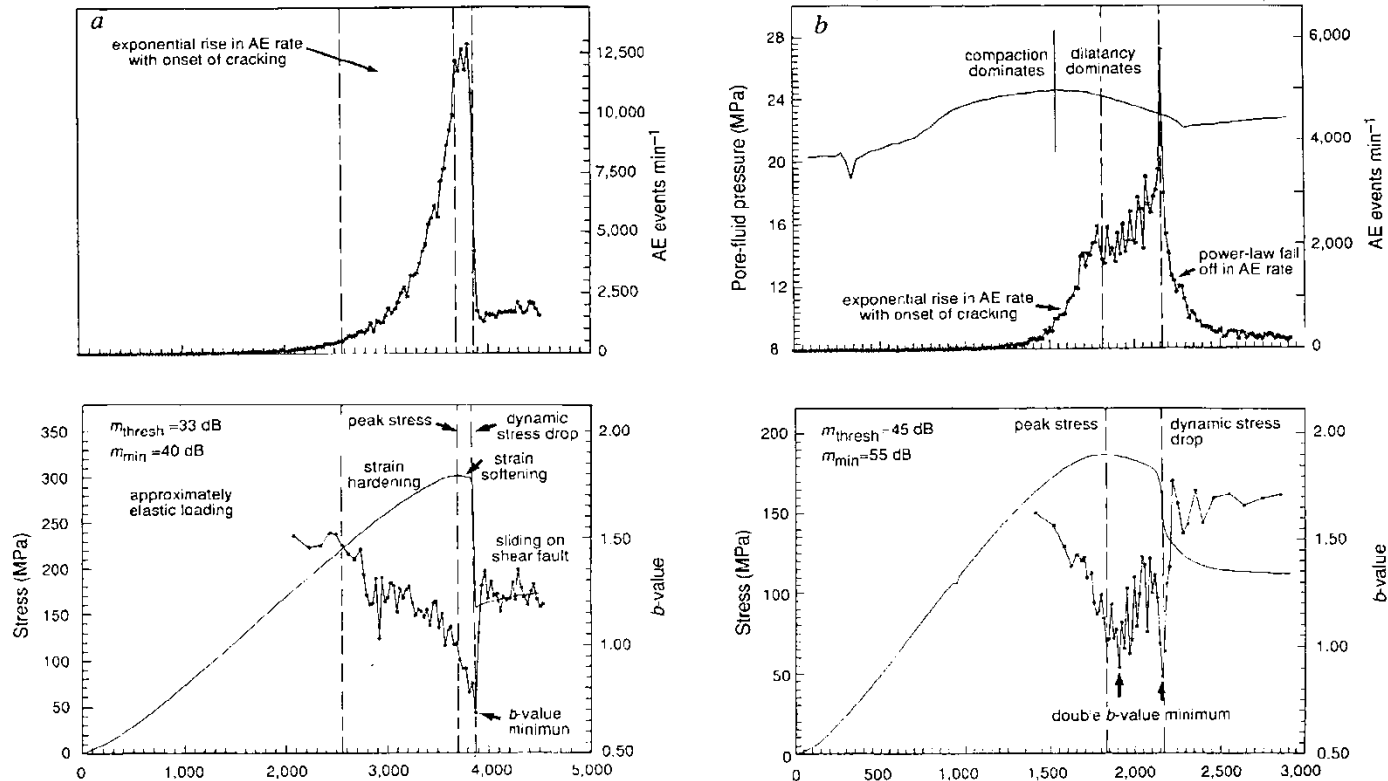


Graham et al., IJRM 2010



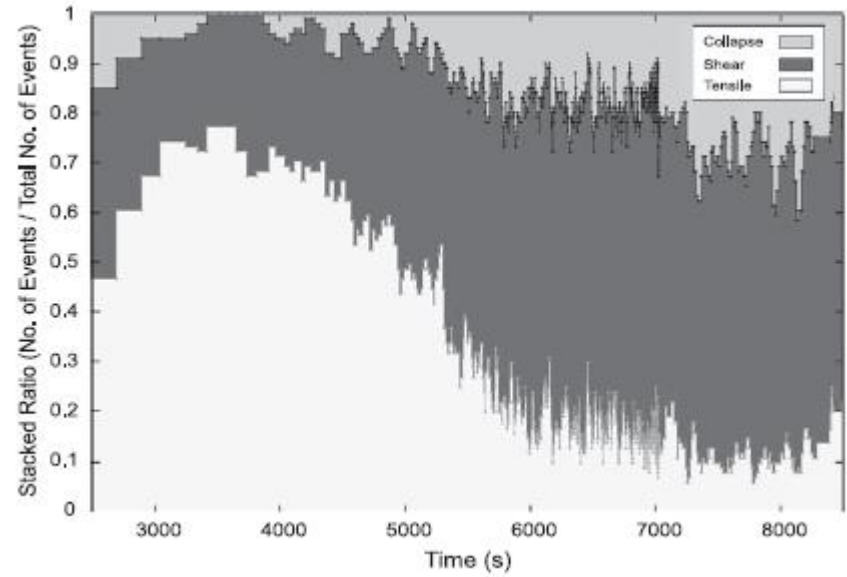
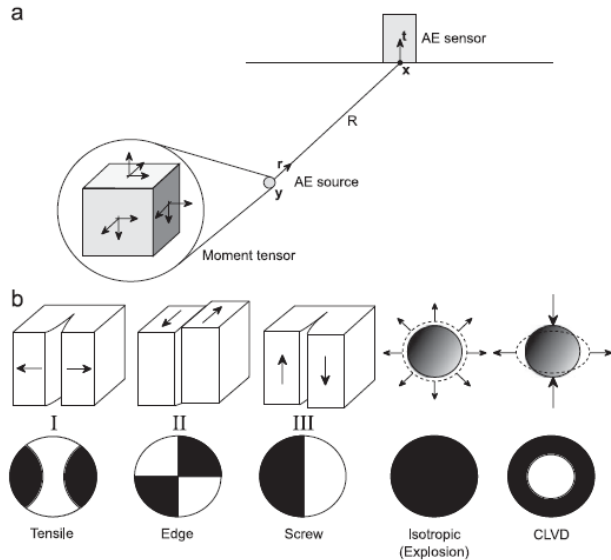
Gutenberg-Richter law:  $\log(N) = a - bM$   
Scaling exponent is the "b-value"

# Acoustic emission event rate and $b$ -value evolution (Sammonds et al, Nature, 1994)



*Drained test:* Event rate increases exponentially or as inverse power law;  $b$ -value drops  
*Undrained:* Produces hiatus due to poro-elastic dilatant hardening

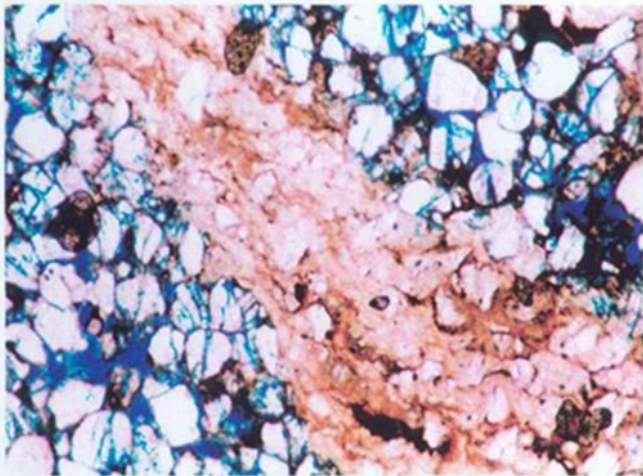
# Partition of Tensor Strain



Graham, Stanchits, Main, Dresen IJRM, 2010

All focal mechanism types occur at all times, consistent with post-test microstructure

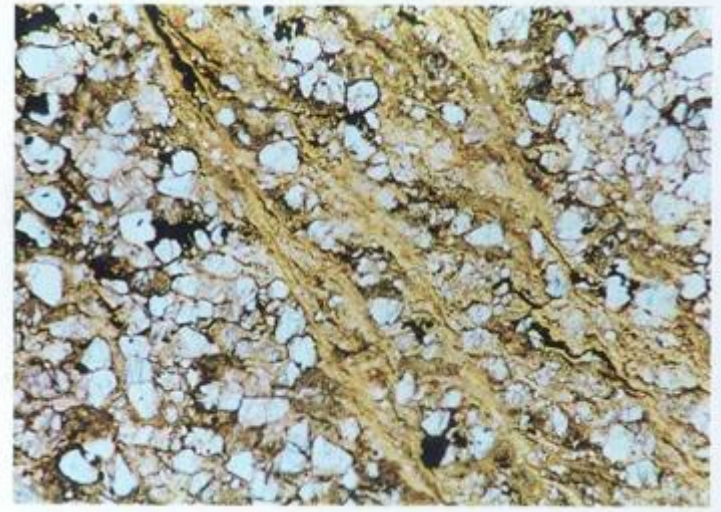
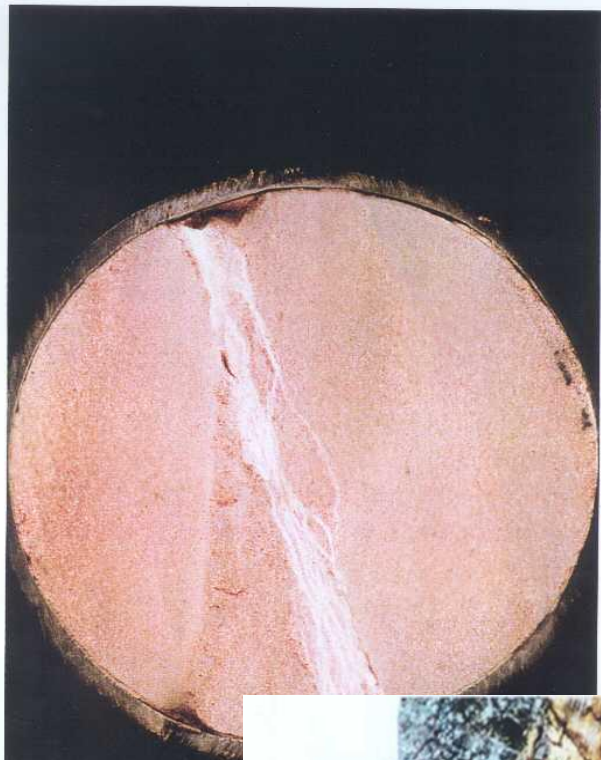
Tensor strain partitions increasingly to shear



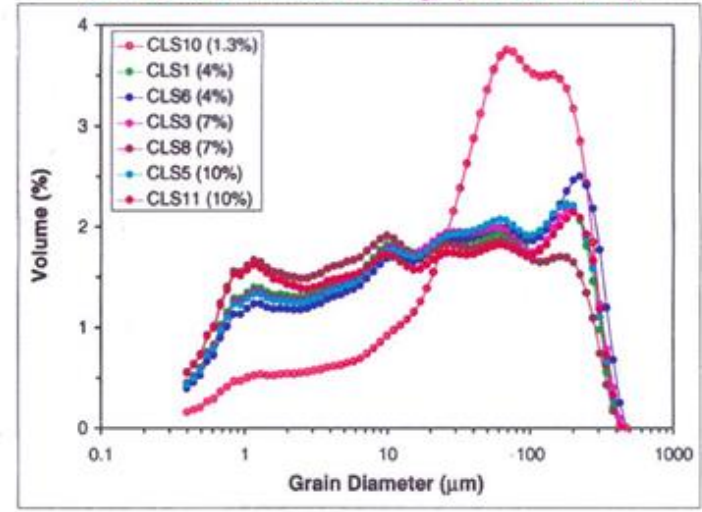
Mair et al., JSG, 2000



# Microstructures

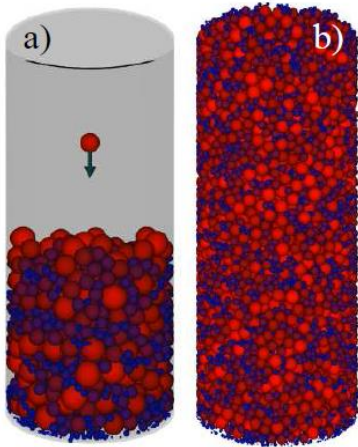


Grain Size Distribution of Gouge (Pe = 34.5 MPa)



# Making a discrete element model

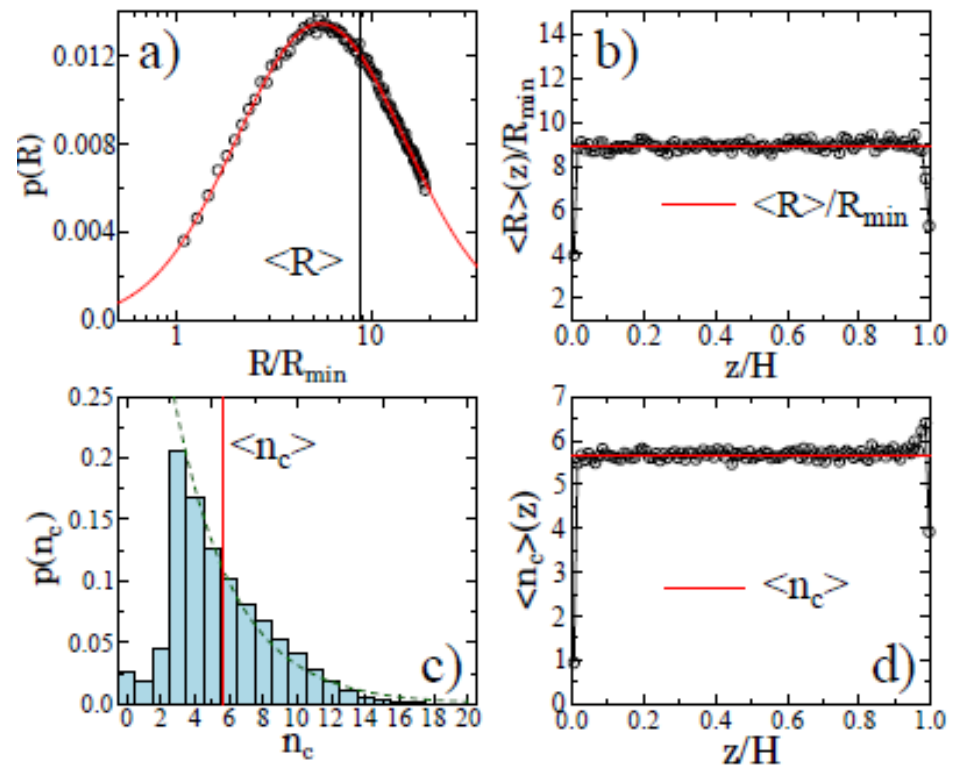
Kun et al., PRE 2013, PRL 2014



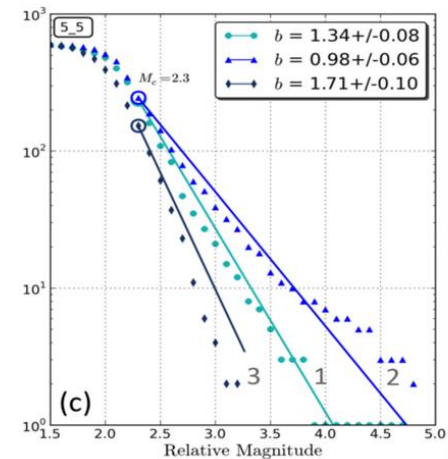
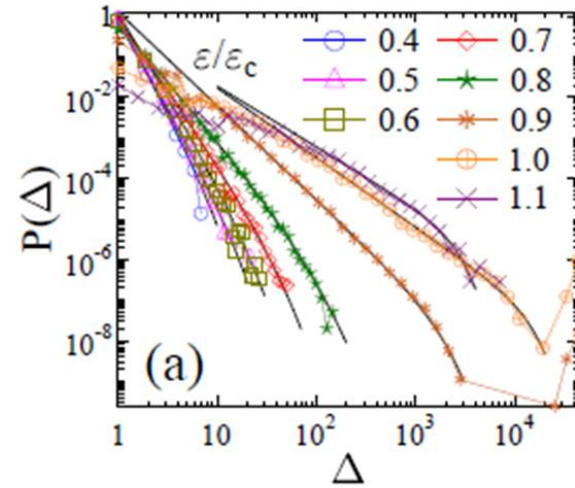
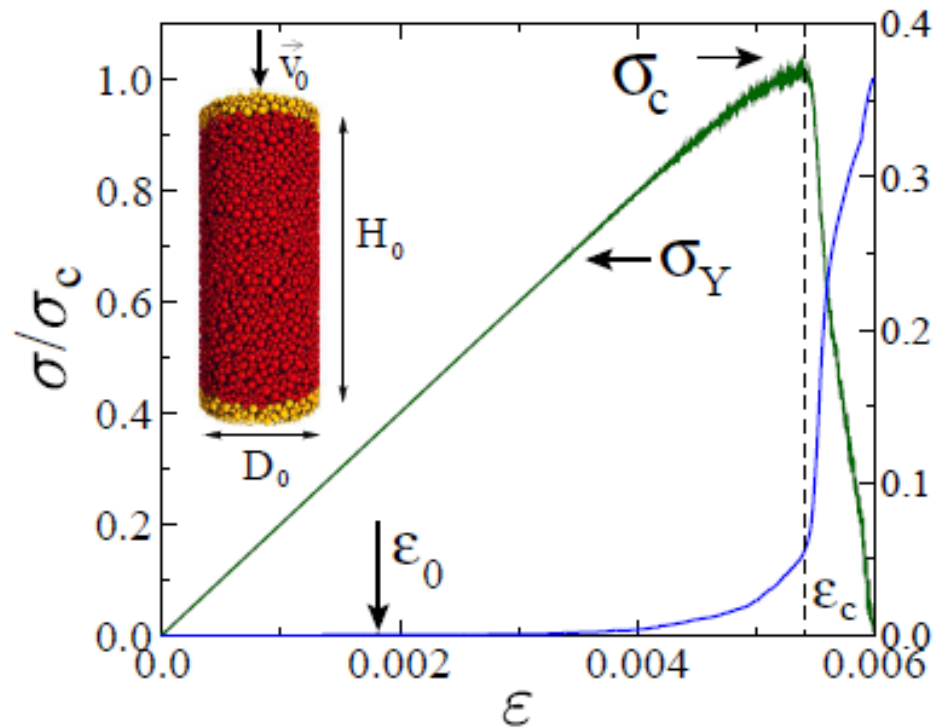
Sedimentation under gravity

- (a) Particle size distribution
- (b) Average grain size vs. vertical position
- (c) Co-ordination number
- (d) Average co-ordination no. vs. depth

- Discrete elements are unbreakable
- Cemented by bonds that deform in tension, compression, shear and bending
- The only disorder is structural
- **No power-laws are put in at step 1**

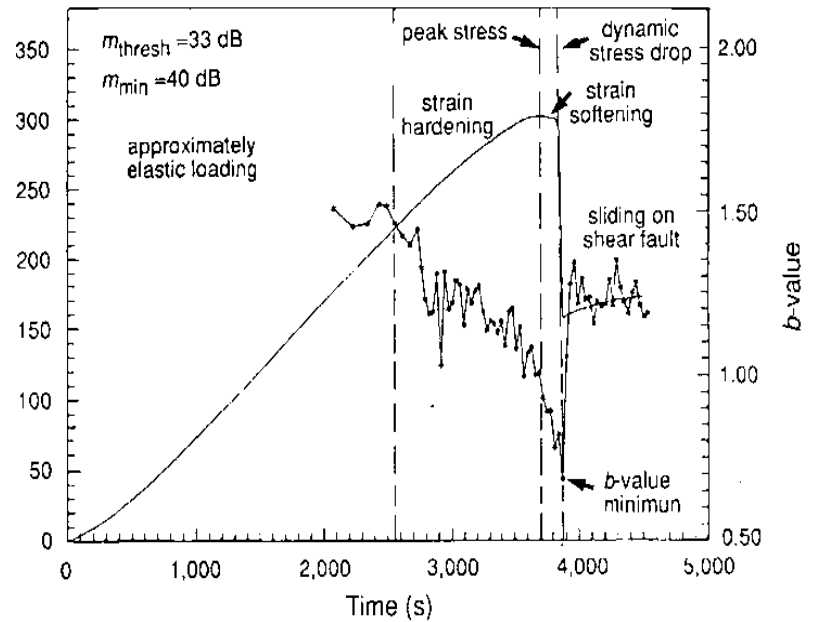
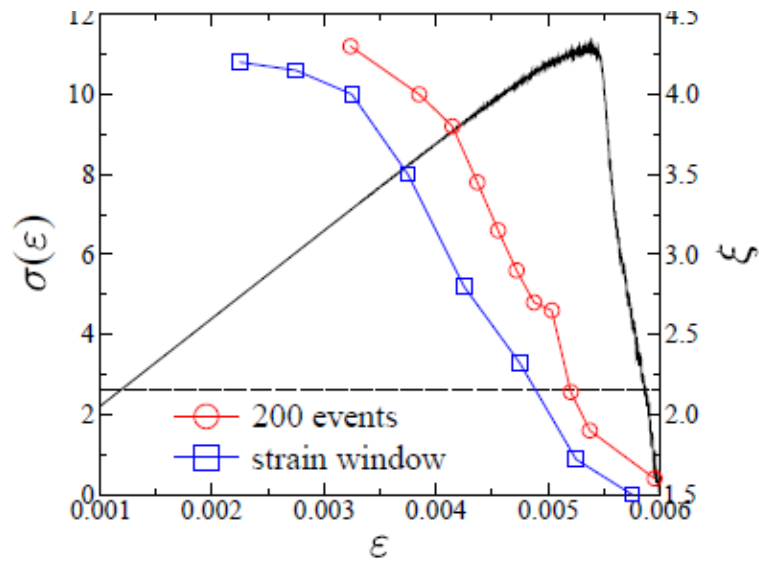
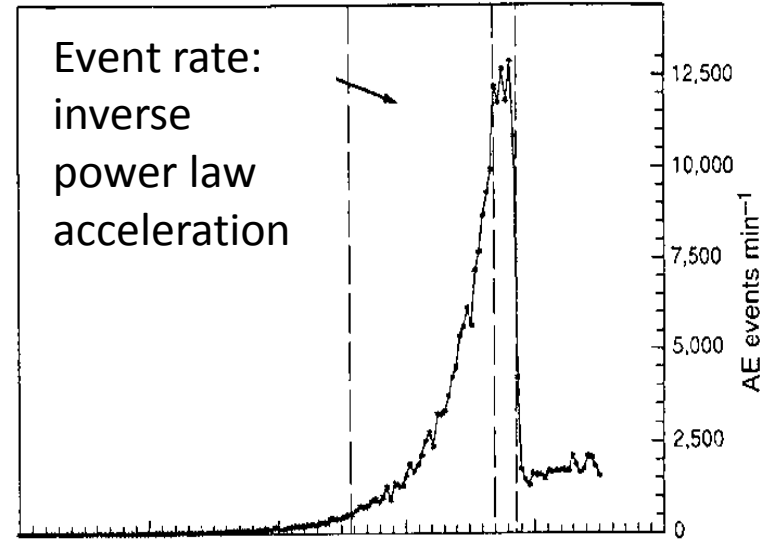
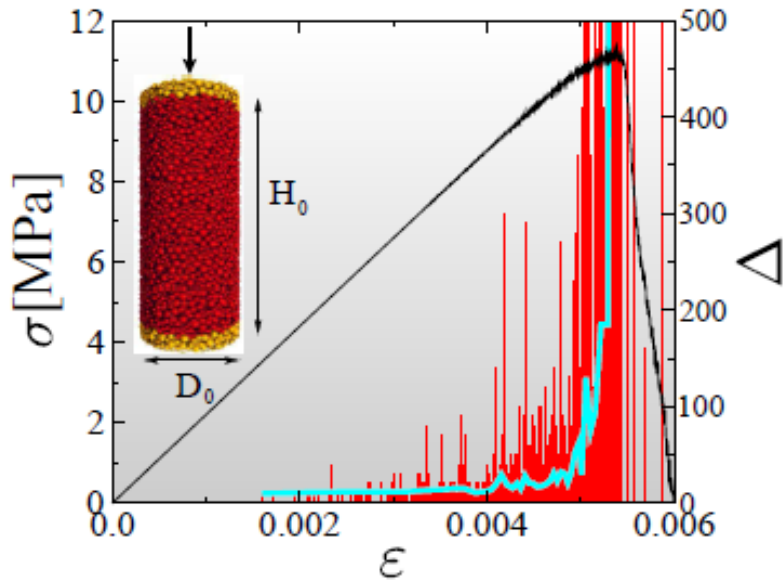


# Squashing the digital rock



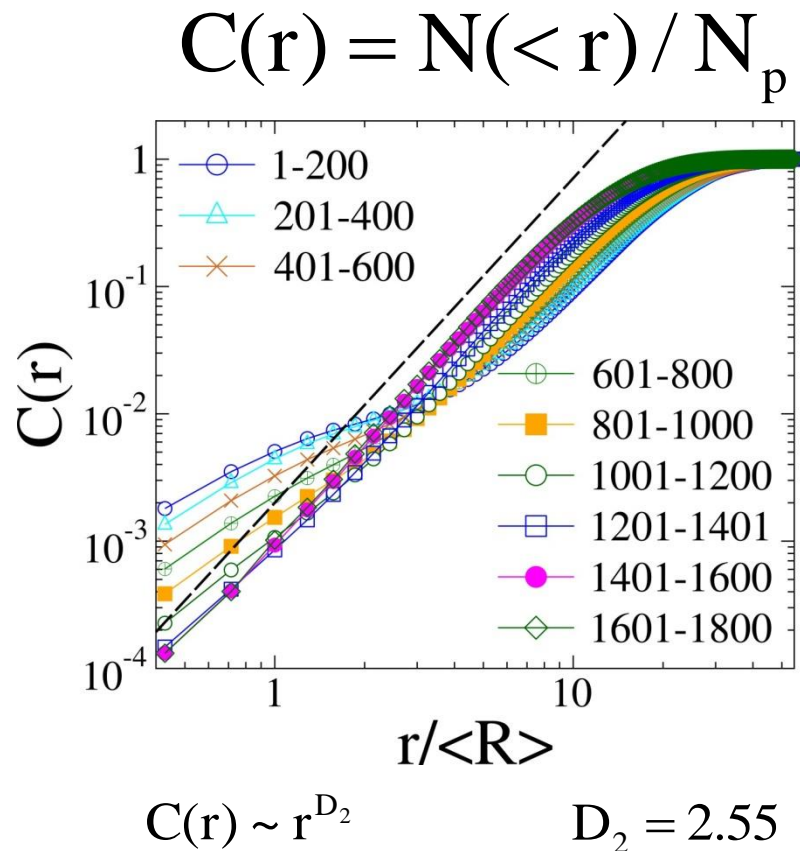


$$\dot{N} = A(1 - t/t_f)^{-p}$$



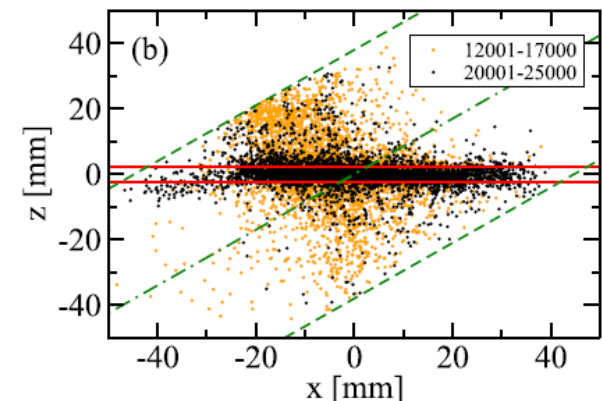
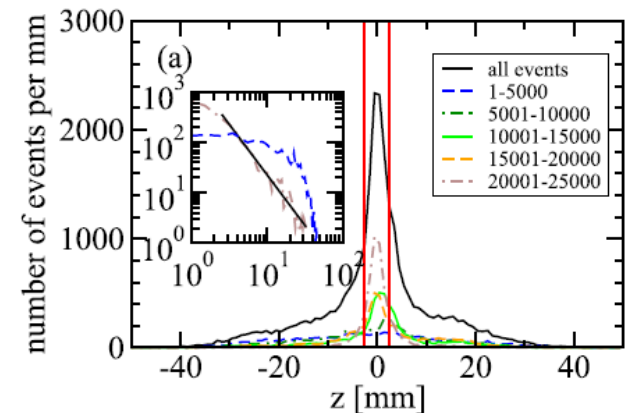
# Approach to failure – spatial localization

Model: Pair correlation function



## Real Data

(Lennartz-Sassinek et al., PRE, 2014)

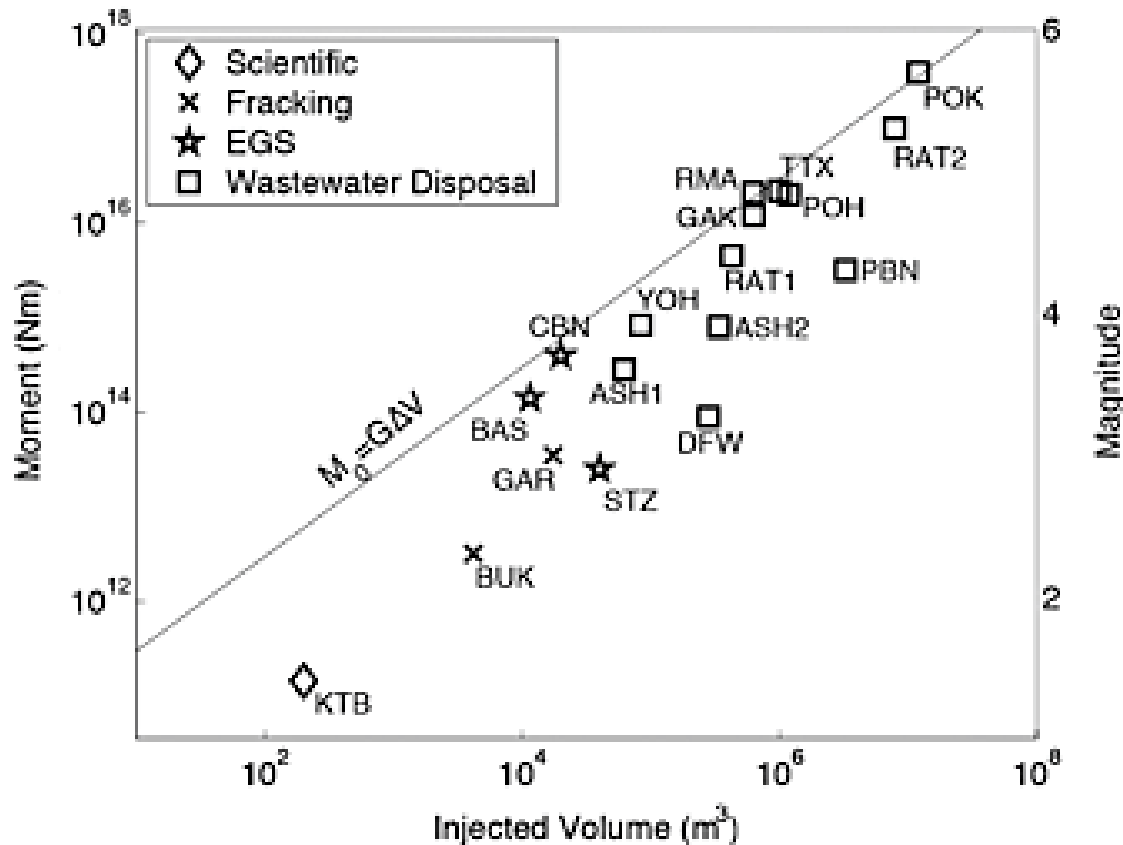


c.f.  $2.25 < D_2 < 2.75$  (Hirata et al., 1987)

# Some published data

Maximum magnitude earthquakes induced by fluid injection (McGarr, 2014)

Note biased sample: "most do not produce earthquakes"



$$F(M) = AM^{-B}$$

$$B = 2b/3$$

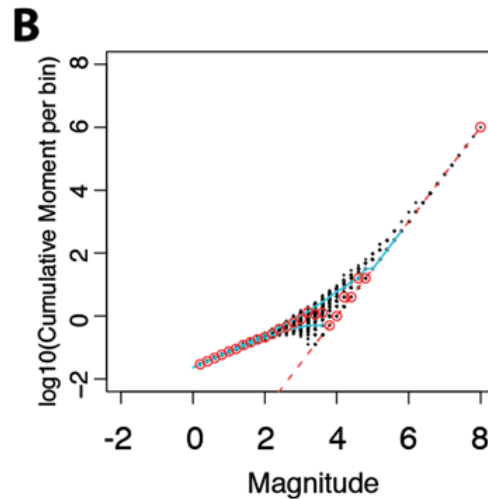
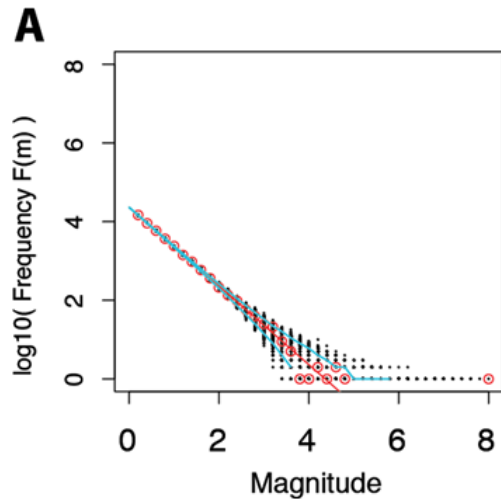
For  $b \cong 1$ ;  $B \cong 2/3$

$$\sum M_i = \left( \frac{B}{1-B} \right) M_{min} N^{\frac{1}{B}}$$

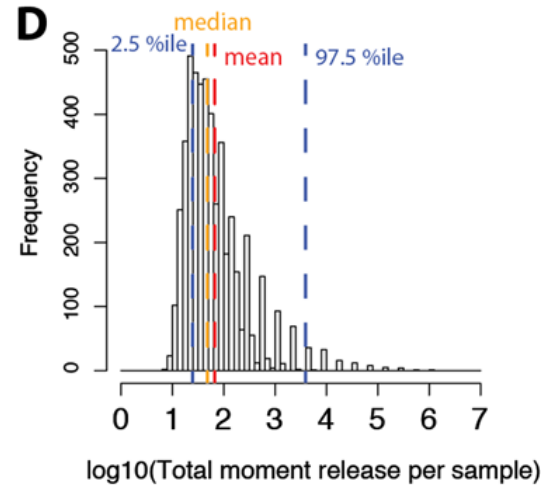
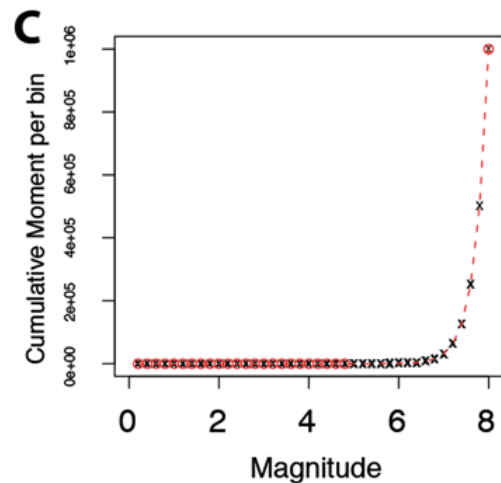
or

$$\sum M_i \cong 2M_{max}$$

# Finite sampling issues



Poisson counting errors propagate into **2 orders of magnitude fluctuations** in total seismic moment released in a given time

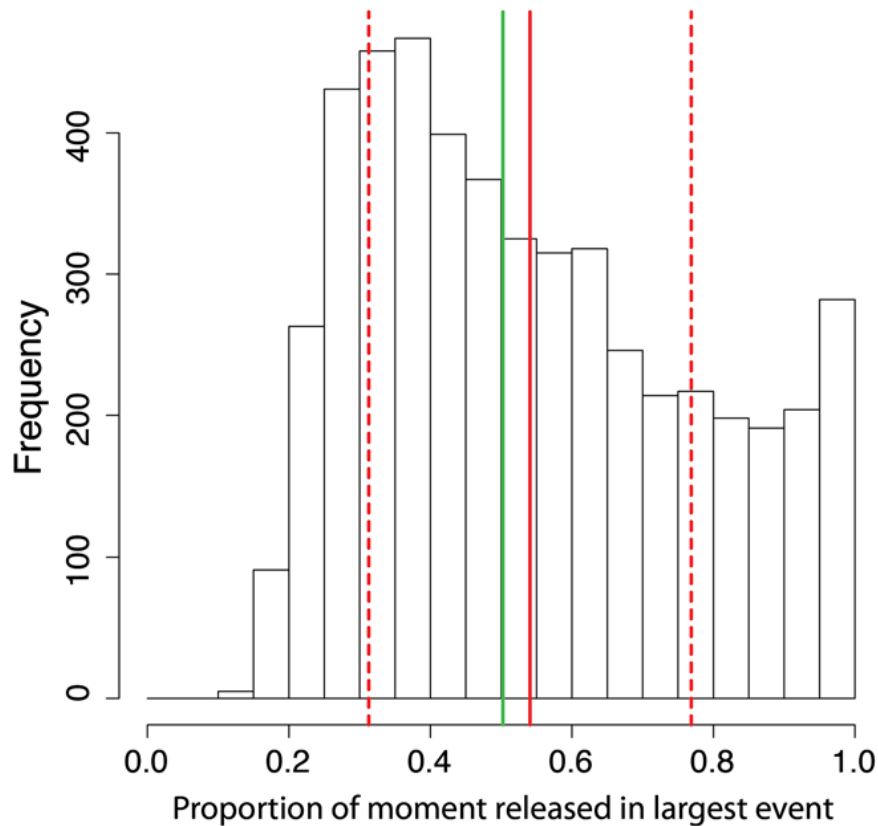


Moment release totally dominated by largest events

Main & Naylor EPJ (2012)

# Uncertainty in assuming

$$\sum M_i \cong 2M_{max}$$



**Red lines:** Mean and standard deviations  $0.54 \pm 0.22$

**Green line:**  $\sum M_i \cong 2M_{max}$



# Evolution of partition factor

We know

$$\sum M_i = \left( \frac{B}{1-B} \right) M_{min} N^{\frac{1}{B}}$$

After yield at  $t=0$ , the partition factor =  $X(t) = \varepsilon^S / \varepsilon^T$  evolves for linear strain rates as

$$X(t) = \frac{\sum M_i(t)}{2\mu V(\varepsilon_0^T + \dot{\varepsilon}t)} = \frac{\left( \frac{B}{1-B} \right) \left( \frac{M_{min}}{2\mu V \varepsilon_0^T} \right) N^{1/B}}{(1 + \dot{\varepsilon}t/\varepsilon_0^T)}$$

For a **transient power-law model** (Main, 2000)

$$N(t) = N_0(1 + t/\tau)^\alpha; \alpha > 0$$

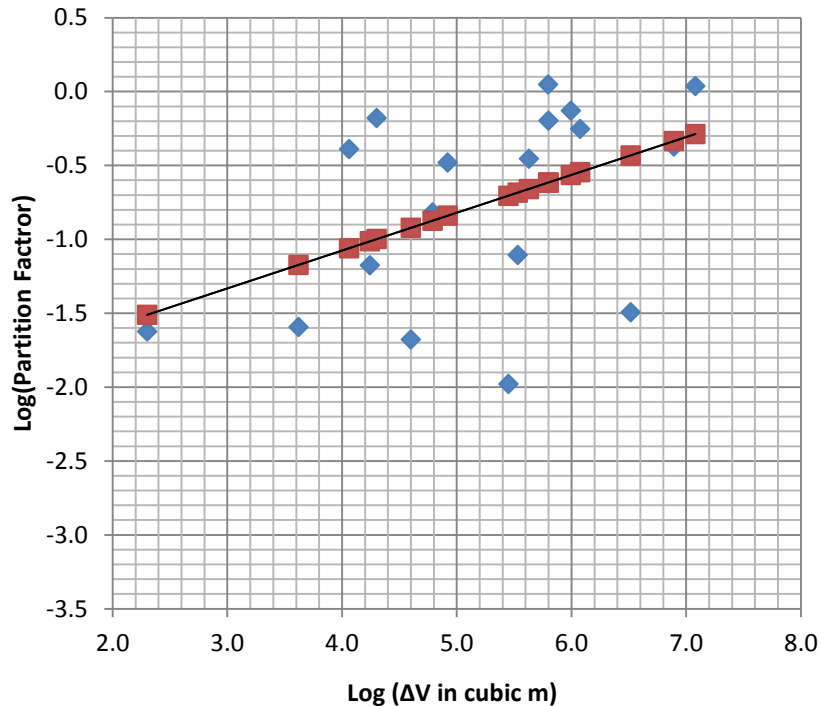
At large strains

$$X(t) = At^{\left(\frac{\alpha}{B}-1\right)}$$

For **inverse power-law growth**, after integration

$$X(t) = A \frac{[1 - (1-t/tf)^{\frac{1-p}{B}}]}{t}$$

# Implied Partition Factor



$$\log(Y) = 0.25(\pm 0.23) \log(\Delta V) - 2.10(\pm 1.26)$$

$$r^2 = 0.217$$

Partition factor increases slowly with strain, as a power law of exponent  $\sim 1/4$

$$\frac{\alpha}{B} - 1 \cong \frac{1}{4}$$

$$\alpha \cong 5/6$$

implies near-linear, slowly decelerating event rate

Large scatter dominates, roughly similar to the finite sampling error – low correlation coefficient

Assumes  $\Delta V = \nabla \dot{V} t$

# Conclusions

- Total strain = elastic + seismic + silent damage or creep
- The location of silent damage can be inferred from changes in hydraulic properties (flow rate), as well as geodetic data
- Discrete element models can now reproduce lab observations, inc. evolution of stress, event rate and  $b$ -value
- (More needs to be done on coupling to pore fluid pressure and add time-dependent weakening)
- In a spatial sample, field data for fluid injection strain partition is consistent with a transient power-law even rate model with  $b \sim 1$ , but
- The transient power-law model is only marginally preferred over one where strain partition is invariant of strain ( $\Delta\text{BIC} \sim 2$ )
- Fluctuations in strain partition for real data (not just the biased sample) exceed finite sampling errors