# Fluid-Induced Seismicity – Comparison of Rate- and State- and Critical Pressure Theory

Friedemann Wenzel

Karlsruher Institut für Technologie

❖ **Abstract** Induced seismicity as generated by the injection of fluid in a homogeneous, porous medium with faults with variable proximity to rupture conditions is simulated using rate- and state-dependent frictional fault theory (RST) of Dieterich (1994) and the critical pressure theory (CPT) developed by Shapiro (2015). In CPT the seismicity is proportional to the pressure rate but limited by the Kaiser Effect. There is no time delay between a change in pressure rate and change in seismicity. RST is more complex and includes a time delay between a pressure change and the seismicity. Comparing both modelling approaches at fixed location this delay can be significant. However, it is small where the seismicity is high and larger where it becomes small. The evolution of the total seismicity in the medium where fluid is injected with time is thus very similar with both modelling approaches so that RST and CPT provide very similar results.

## ❖ Essentials of the Critical Pressure Theory (CPT)

fault density: $n_f(\vec{x})$

probability that the criticality parameter is less than C: $F_c(C)$

pressure due to injection: $p(\vec{x},t)$

Seismicity rate:

$$N(t) = \int n_f(\vec{x}) \cdot f_c(p(\vec{x},t)) \cdot \frac{\partial p(\vec{x},t)}{\partial t} \cdot dV$$

$\frac{\partial p(\vec{x},t)}{\partial t}$ must be positive and the Kaiser Effect considered. For constant volume rate and homogeneous diffusion

$$p(r,t) = \frac{\dot{Q}_0}{4\pi D S_r} \cdot \left\{1 - erf\left\langle \frac{r}{\sqrt{4Dt}} \right\rangle\right\}$$

Seismicity rate is:

$$N(t) = \frac{n_f}{C_2 - C_1} \cdot \frac{\dot{Q}_0}{S} = N_0$$

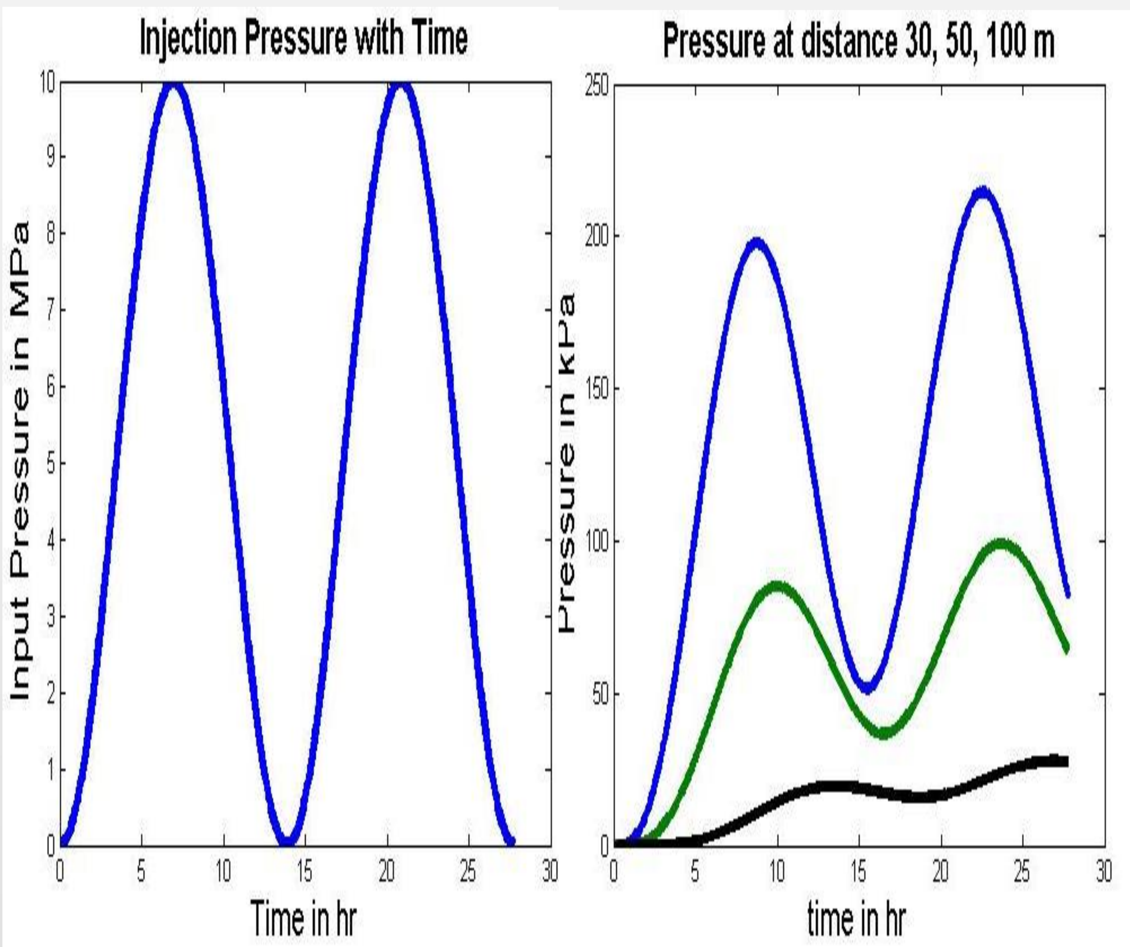


*Fig. 1: Left panel shows injection pressure (over hydrostatic) at the origin with two cycles of about 14 h each and 10 MPa amplitude. The right panel show pressure at 30, 50, and 100m calculated with a diffusion constant of $D = 0.1 m^2/s$.*

## ❖ The Rate- and State-dependent Theory (RST)

Dieterich's (1994) formula: Seismicity rate in response to shear and normal stress changes:

$$N(t) = \frac{1}{\gamma(t)} \cdot \frac{v_{tec}}{\dot{\tau}_{tec}}$$

$$\frac{d\gamma}{dt} = \frac{1}{A \cdot \sigma_n(t)} \cdot \left\{1 - \gamma(t) \cdot \left\{\frac{\tau(t)}{\sigma_n(t)} - \alpha\right\} \cdot \frac{d\sigma_n}{dt}\right\}$$

background seismicity: $v_{tec}$

natural stress rate: $\dot{\tau}_{tec}$

If normal stress changes due to fluid injection:

$$\sigma_n(t) = \sigma^0{}_n - p(t)$$

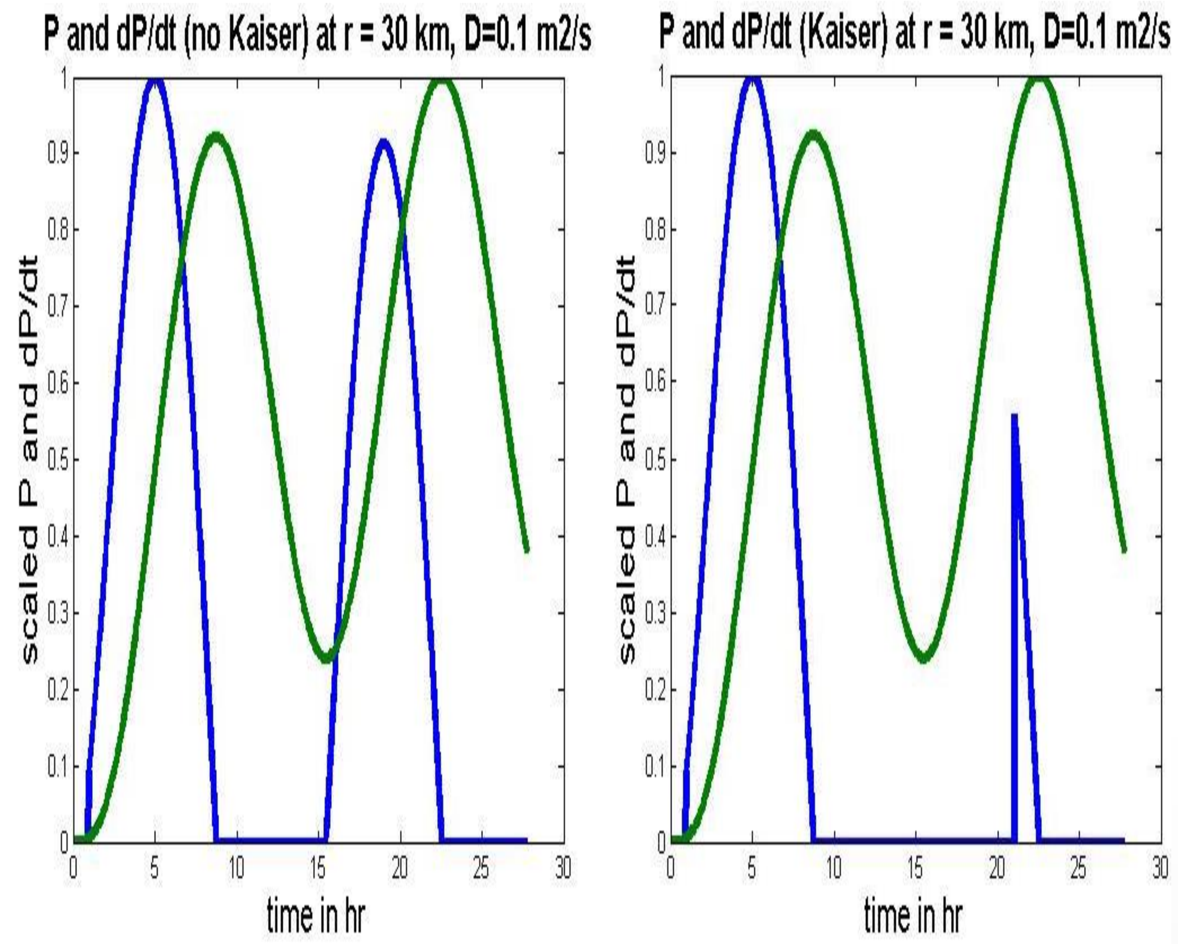


*Fig. 2: Pressure and temporal pressure derivative at 30m distance, scaled to relative amplitudes so that they fit into one plot. In the left panel the derivative is set to zero only if it becomes negative. The right panel considers the Kaiser Effect and keeps $\dot{p}(r,t)$ at zero level until the pressure in the second cycle exceeds the maximal value it experienced before.*

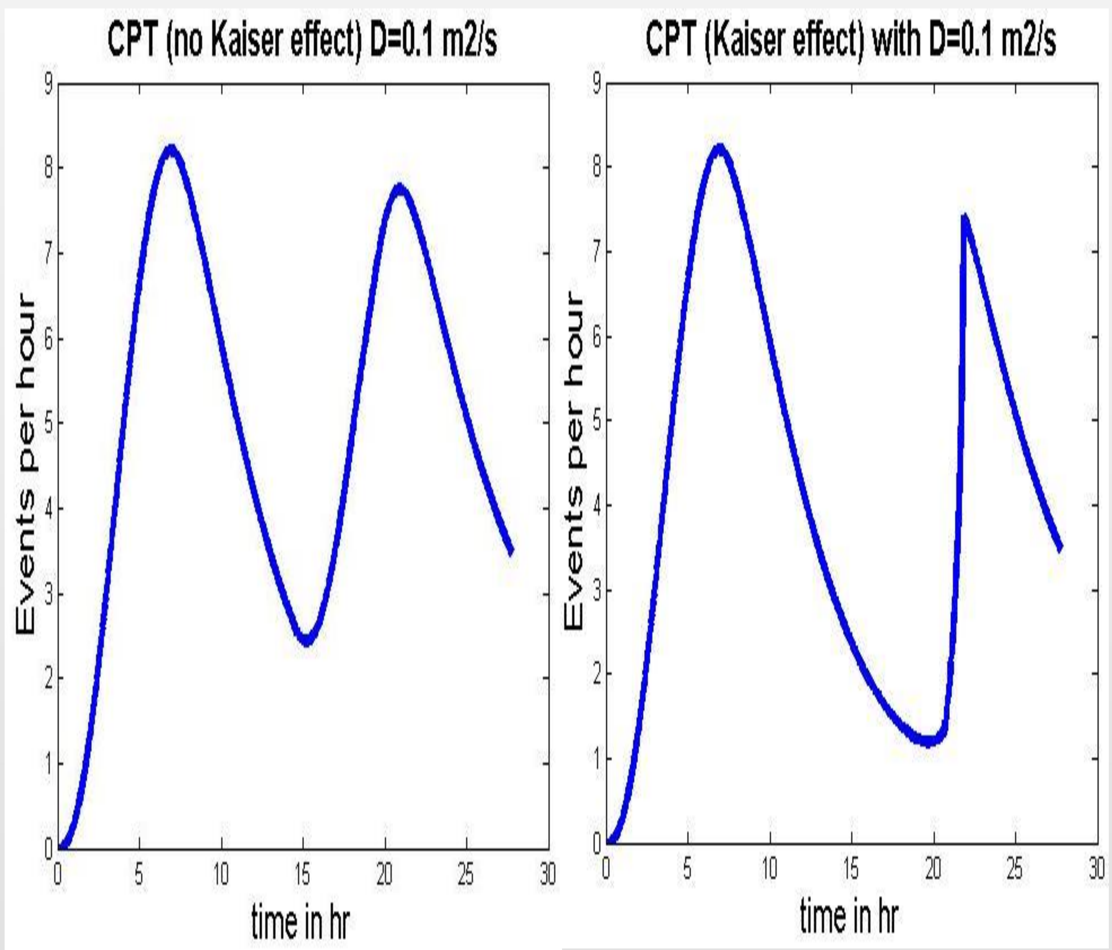


*Fig. 3: Left: Seismicity evolution with CPT ignoring the Kaiser Effect. Note that it shows almost identical periodicity as the injection pressure. Right: Seismicity evolution with CPT and considering the Kaiser Effect. The early (until 15 h) and late (after 22 h) seismicity are identical to the one shown on the left. Its appearance between 15 and 22 h is strongly affected by the Kaiser Effect.*

$$\frac{d\gamma}{dt} = \frac{1}{A \cdot \sigma^0{}_n}\left\{1 + \frac{p(t)}{\sigma^0{}_n} - \gamma(t)\right.$$

$$\left.\cdot\left\{\dot{\tau}_{tec} \cdot \left\{1 + \frac{p(t)}{\sigma^0{}_n}\right\} + g_0 \cdot \dot{p}(t)\right\}\right\}$$

$$g_0 = \frac{\tau_{tec}}{\sigma^0{}_n} - \alpha$$

Or written as Ricatti equation

$$N(t) = R(t) \cdot v_{tec}$$

$$b(t) = \frac{\dot{\tau}_{tec}}{C_1} \cdot \left\{1 + \frac{p(t)}{\sigma^0{}_n}\right\}$$

$$a(t) = b(t) + \frac{g_0}{C_1} \cdot \dot{p}(t)$$

$$R(t) = \frac{1}{exp\left[-\int_0^t a(\vartheta)d\vartheta\right] + \int_0^t b(\tau) \cdot exp\left[-\int_\tau^t a(\vartheta)d\vartheta\right]d\tau}$$

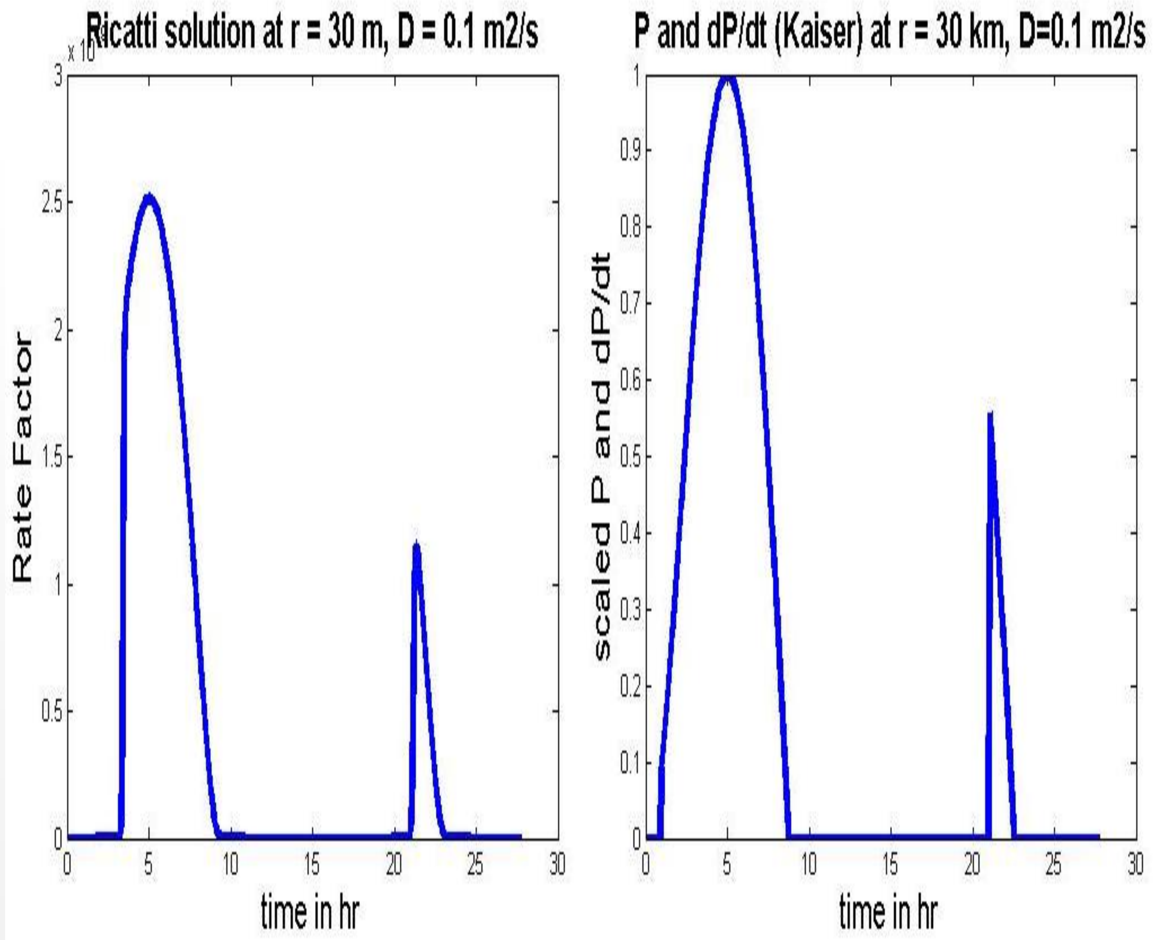


*Fig. 4: Solution of Ricatti Equation (11) at distance 30 m (left) and comparison with the CPT ansatz that includes the Kaiser Effect (right).*

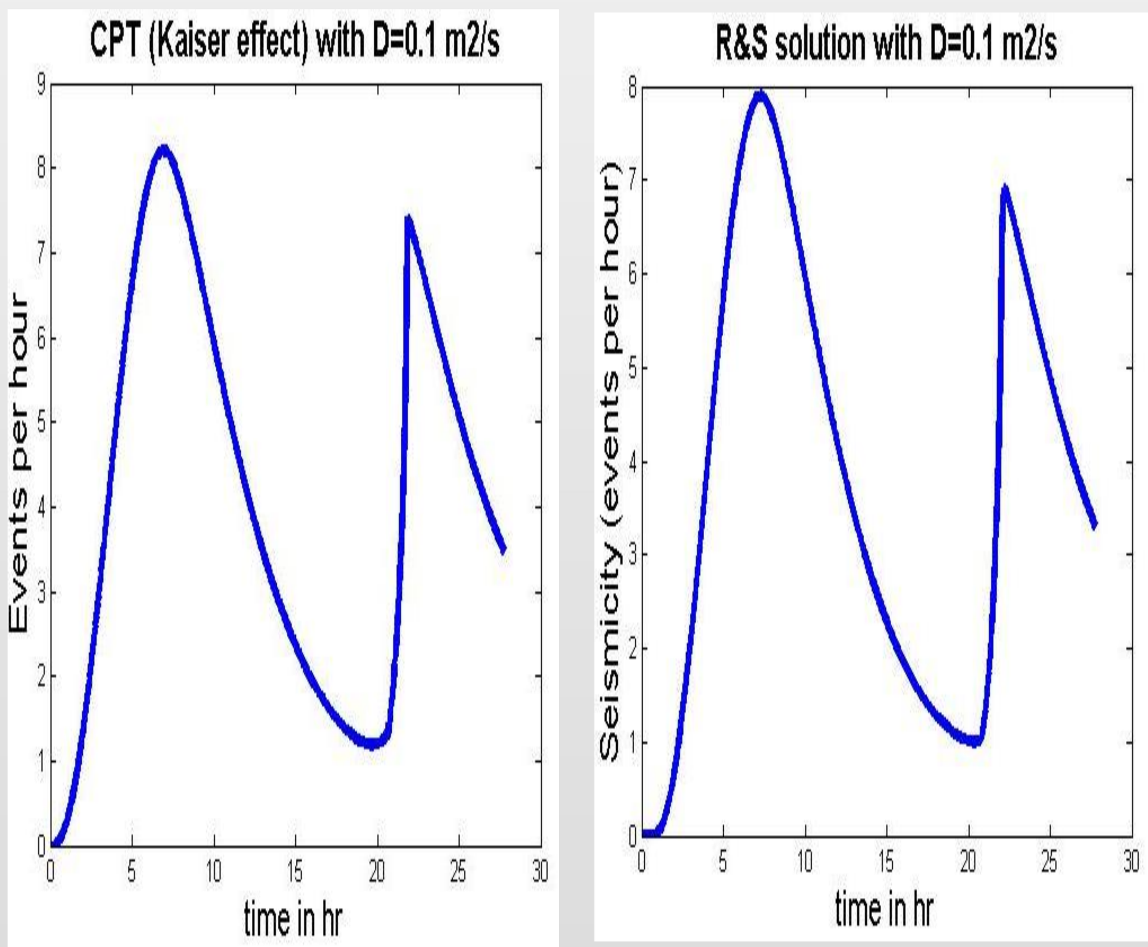


*Fig. 5: Comparison between the CPT and RST results for total seismicity of the two-cycle injection with D=0.1 m2/s and lower trigger threshold of 1 kPa after integration over the entire volume of fluid infiltration. There are no visible differences.*

## ❖ Reference

J. Dieterich, A Constitutive Law for the Rate of Earthquake Production and its Application to Earthquake Clustering, J. Geophys. Res., vol. 99(B2), 2601-2618, 1994.

S.A. Shapiro, Fluid-Induced Seismicity, Cambridge University Press, p.75. 2015

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*Geophysical Institute*
*Friedemann.wenzel@kit.edu*