Sequential Data Assimilation for Seismicity

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Summary

PSHA is dominated by statistical approaches, where it has been a challenge to include more physics-based information. We adopt the statistical framework of sequential data assimilation - extensively developed for weather forecasting - to efficiently integrate observations and prior physical knowledge, while acknowledging errors in both sources of information. To proof this concept we perform a perfect model test in an analogue subduction zone to probabilisti-cally estimate the current and future state of stress and strength on the megathrust interface.

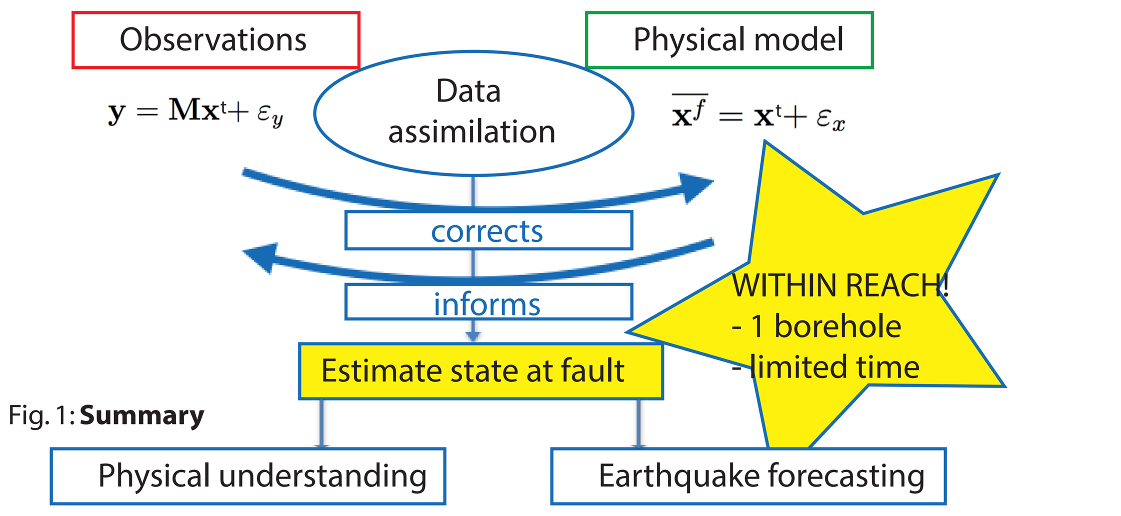

Fig. 1: **Summary**

A quantitative and qualitative evaluation shows that meaningful information on the stress and strength is available, even when only data from a single borehole is assimilated over only a part of a seismic cycle. This is possible, since the sampled error covariance matrix contains prior information on the physics that relates velocities, stresses, and pressures at the surface to those at the fault. During the analysis step, stress and strength distributions are thus re-constructed to either inhibit or trigger events. In the subsequent forward propagation step the physical equations are solved to propagate the updated states forward in time and thus pro-vide probabilistic information on the occurrence of the next analogue earthquake. The sys-tems forecasting ability turns out to be significantly better than using a periodic model to forecast the large events in this quasi-periodic sequence (e.g., requiring an alarm ~17% vs. ~68% of the time to forecast 70% of 21 events correctly). Although several challenges for applications to natural data remain, we believe this first step provides an alternative vision on how to statistically combine data and prior physical knowledge.

Sequential data assimilation is ...

A Bayesian framework to include temporal data and its error into a physical model to estimate hidden, dynamic state variables (and parameters) (Evensen, 2009).

An Ensemble Kalman Filter is simple and efficient and works well for high-dimensions.
1. Propagate many models (~ as you have them!) using prior physical knowledge
2. Update using data misfit and covariances in least-square solution of Bayes' theorem:

Kalman gain

$$\mathbf{x}_n^a = \mathbf{x}_n^f + \mathbf{C}_{xx}^{f,e}\mathbf{M}^T(\mathbf{M}\mathbf{C}_{xx}^{f,e}\mathbf{M}^T + \mathbf{C}_{yy}^e)^{-1}(\mathbf{y}_n - \mathbf{M}\mathbf{x}_n^f)$$

Posterior Prior Covariance links data to state; Weights for model + data error; Data misfit

Eq. 1: Ensemble Kalman Filter

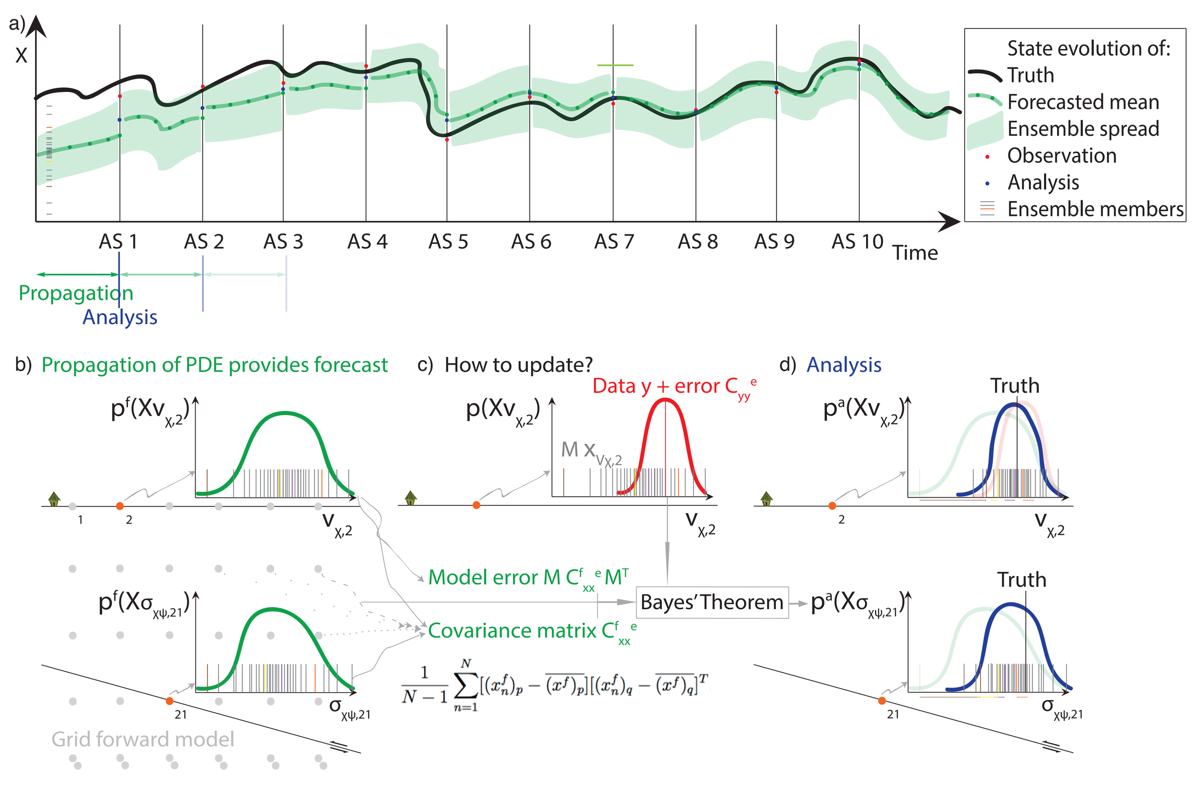

Fig. 2: **Cartoon explaining Ensemble Kalman Filters** in a) time, highlighting how ensembles a) forecast the prior, b) relate to the data, and d) form the analysis.

Method for Proof of concept

Sequential data assimilation has not been applied to estimate states relating to seismicity, although pioneering studies use statistical models (Werner et al., 2011) and scenario-based, off-line approaches (e.g., Hori et al, 2014). Hence a proof of concept is required. That requires a perfect model test in which synthetic data is taken from an additional, numerical model, which represents the truth.

We sequentially assimilate noised, synthetic velocity, stress, and pressure data from a single location in a simplified subduction setup (Fig. 3a, Corbi et al., 2013). Current state-of-the-art errors are downscaled and applied.

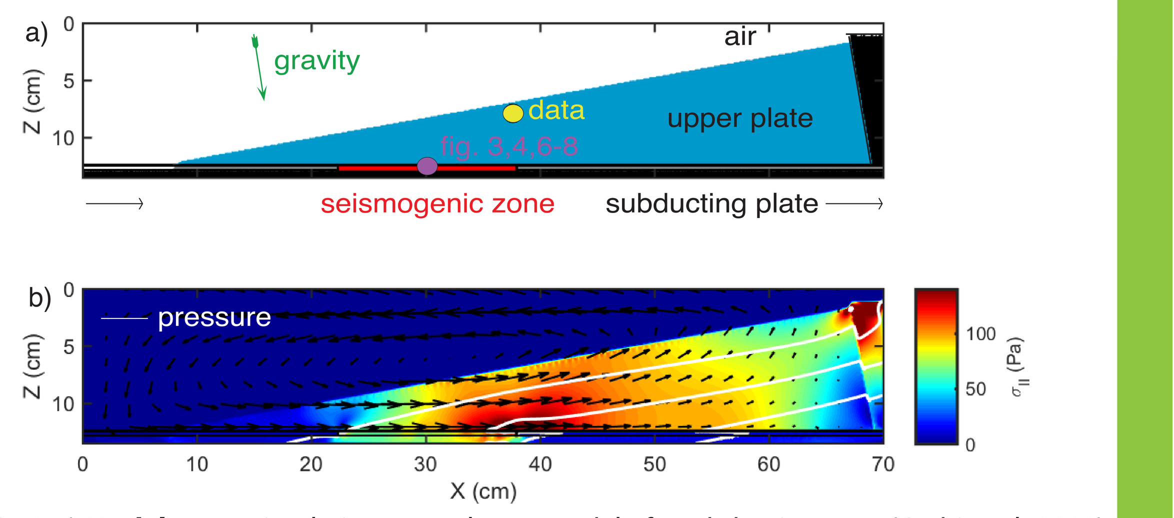

Fig. 3: a) **Model setup** simulating an analogue model of a subduction zone (Corbi et al., 2013). b) Typical distribution of the estimated five state variable types.

Using an Ensemble Kalman Filter (eq. 1) we update 150 ensemble members of a Partial Differential Equation-driven seismic cycle model (STM; van Dinther et al., JGR, 2013a,b). This visco-elasto-plastic continuum forward model solves Navier-Stokes equations with a rate-dependent friction coefficient (eq. 2). To estimate fault slip or plastic yielding, we thus need to estimate five state types in green:

$$\frac{\partial v_\chi}{\partial \chi} + \frac{\partial v_\psi}{\partial \psi} = 0,$$

$$\frac{\partial \sigma'_{\chi\chi}}{\partial \chi} + \frac{\partial \sigma'_{\chi\psi}}{\partial \psi} - \frac{\partial P}{\partial \chi} = \rho \frac{D v_\psi}{Dt},$$

$$\frac{\partial \sigma'_{\psi\chi}}{\partial \chi} + \frac{\partial \sigma'_{\psi\psi}}{\partial \psi} - \frac{\partial P}{\partial \psi} = \rho \frac{D v_\psi}{Dt} - \rho g$$

$$\dot{\epsilon}'_{ij} = \frac{1}{2\eta_v}\sigma'_{ij} + \frac{1}{2G}\frac{D\sigma'_{ij}}{Dt} + \begin{cases} 0 & \text{for } \sigma'_{II} < \sigma_{\text{yield}} \\ \lambda \frac{\sigma'_{ij}}{2\sigma'_{II}} & \text{for } \sigma'_{II} = \sigma_{\text{yield}} \end{cases}$$

$$\sqrt{\sigma'^2_{\chi\chi} + \sigma'^2_{\chi\psi}} \geq C + \mu(v_\chi, v_\psi)P$$

$$\mu = \mu_s(1-\gamma) + \mu_s \frac{\gamma}{1 + \frac{V}{V_c}}$$

Eq. 2: Conservation and constitutive equations forward model

Can we capture stress to fit synthetic analogue seismicity?

Yes, we can! Even through assimilating a single set of interseismic borehole data, shear stresses distribution can be recovered really well (Fig. 4).

Probabilistic estimates of fault stress and dynamic strength evolution capture the truth exceptionally well (Fig. 5, 7).

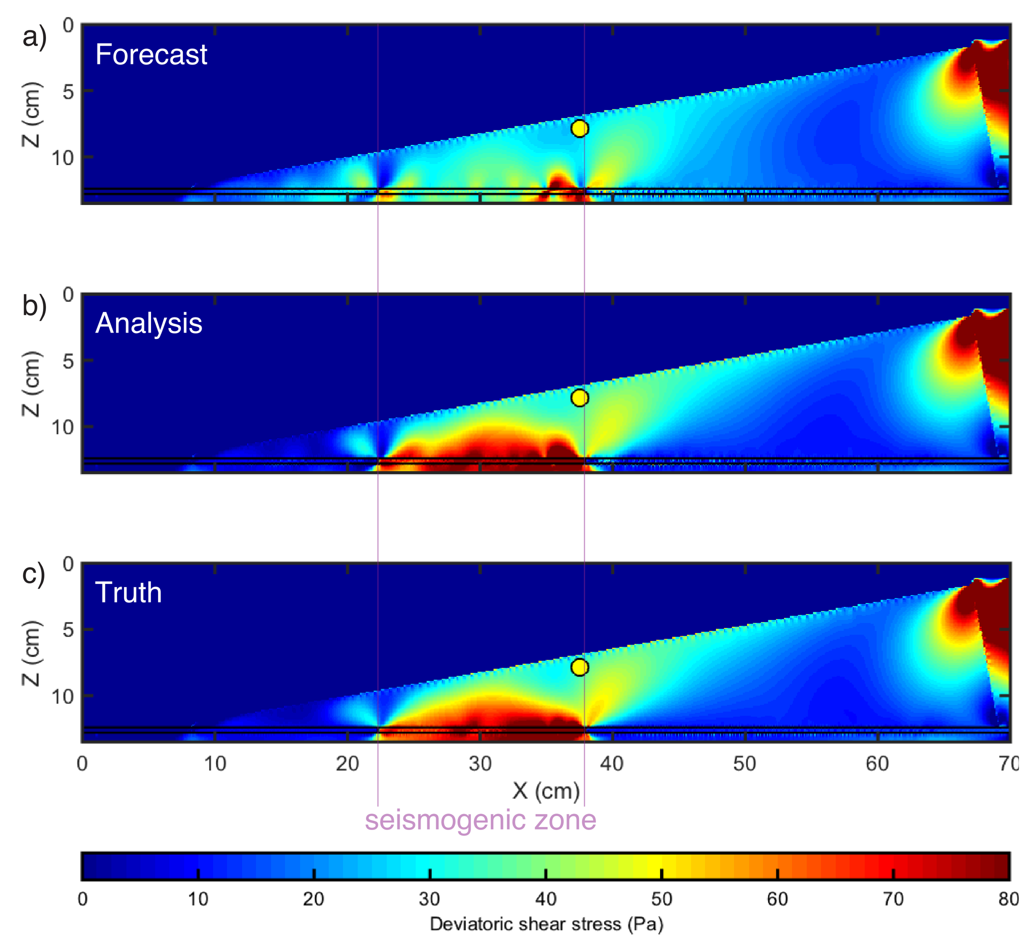

Fig. 4: **Spatial shear stress recovery.**
During the assimilation, data from the borehole at the yellow dot in the synthetic data model in c) has been added to the postseismic forecast in a) to update to b).

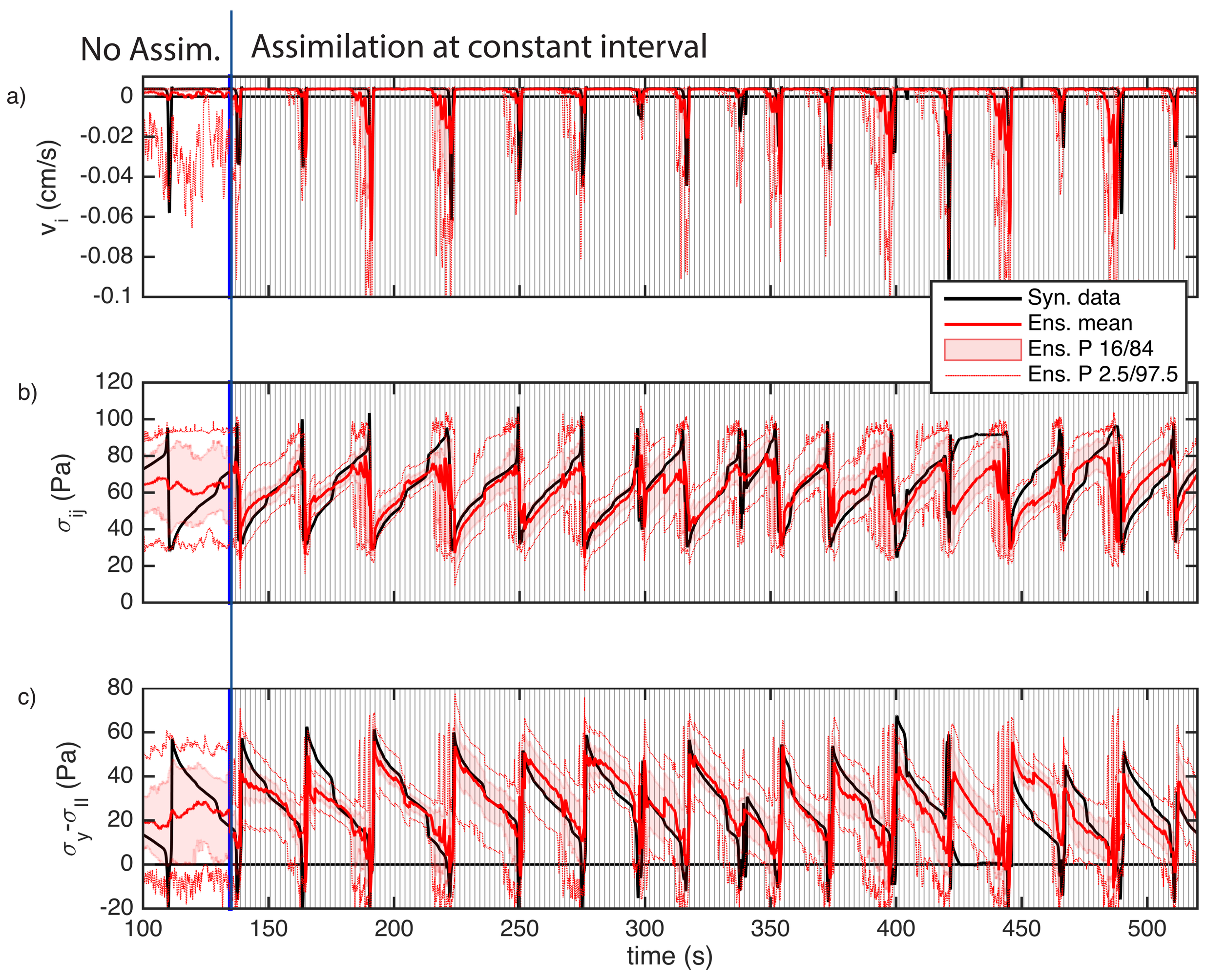

Fig. 5: **State evolution in center seismogenic zone.**
After assimilation starts, ensemble statistics (red) track the black truth remarkably well. Especially event timing, as average levels are known from known parameters.

Can we forecast events?

Yes, we can! The systems forecasting ability turns out to be significantly better that of a periodic recurrence model to forecast the large events in this quasi-periodic sequence (requiring an alarm ~17% vs. ~68% of the time to forecast 70% of 21 events) (Fig. 6).

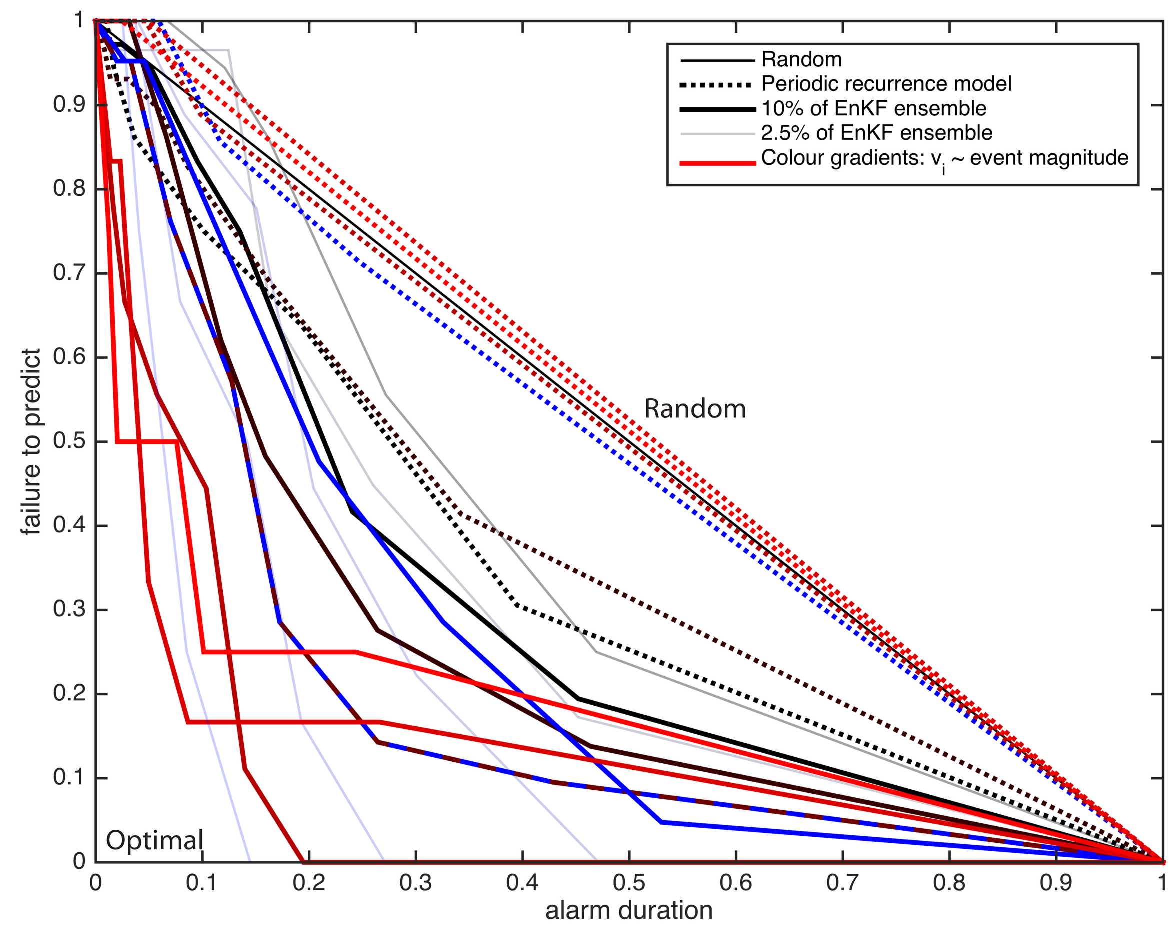

Fig. 6: **Error diagram to assist decision making on earthquake forecasting.**
An alarm sounds when six different velocity thresholds (~event sizes) are passed (different lines). We use two different percentages of the assemble to sound an alarm (solid black to red, when increasing size, and transparent black to blue).

Why does this work? How stress and strength at a fault estimated?

The prior information from the physical model provides the content of the sampled, model error covariance matrix, which contains the information on how to relate velocities, stresses, and pressure at the surface to those at the fault and throughout the medium. Velocities and stresses at the surface and at the fault thus covary enough for the Ensemble Kalman Filter to provide a meaning full update, despite very large stress data errors. A one point update shows the update for one observation follows the least-square fit between (Fig. 8). A spatially smoothly covarying pattern illustrates why data from a single location is enough (Fig. 9).

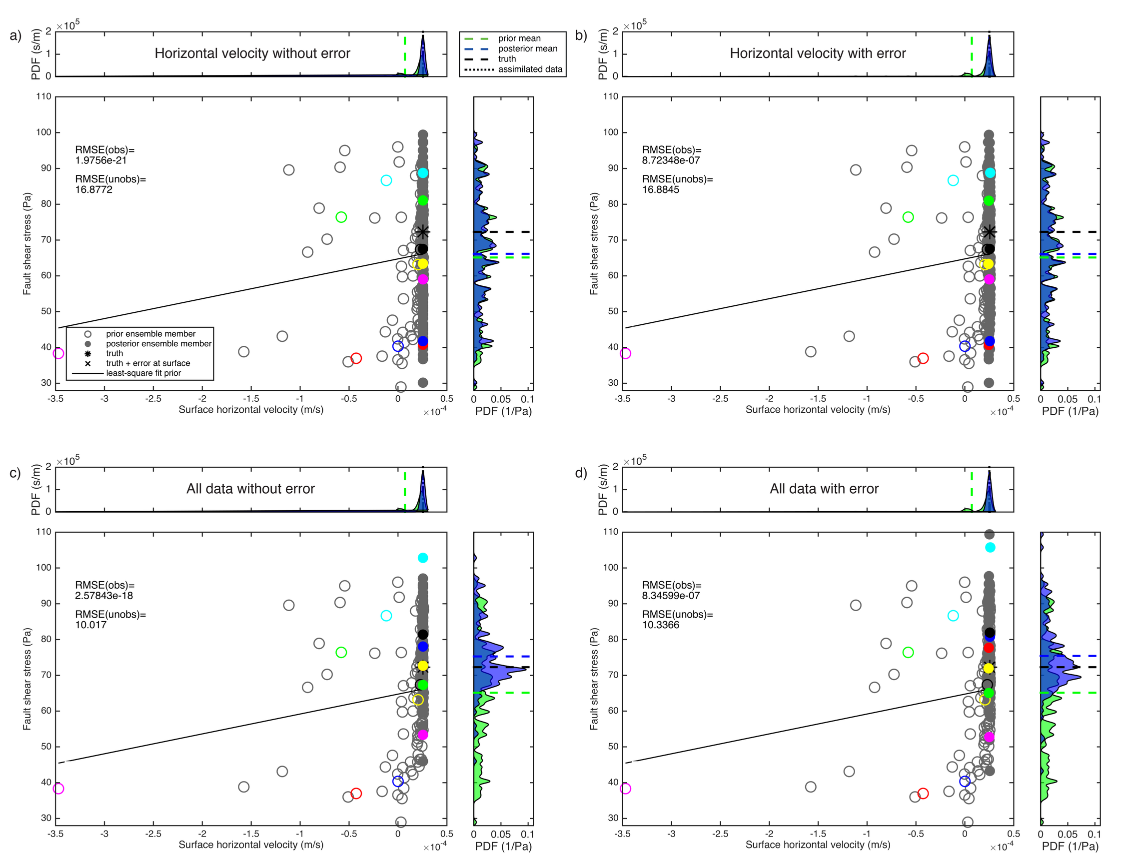

Fig. 8: **How to update at a hidden state (y) from one observed state (x)?**
Ensemble members visualize correlation at one element of covariance matrix Cxx.
For understanding the complexity of the assimilated data is increased from a) only horizontal velocity, without error, to b) include an error, to c) to add the other 4 types, and d) with an error.

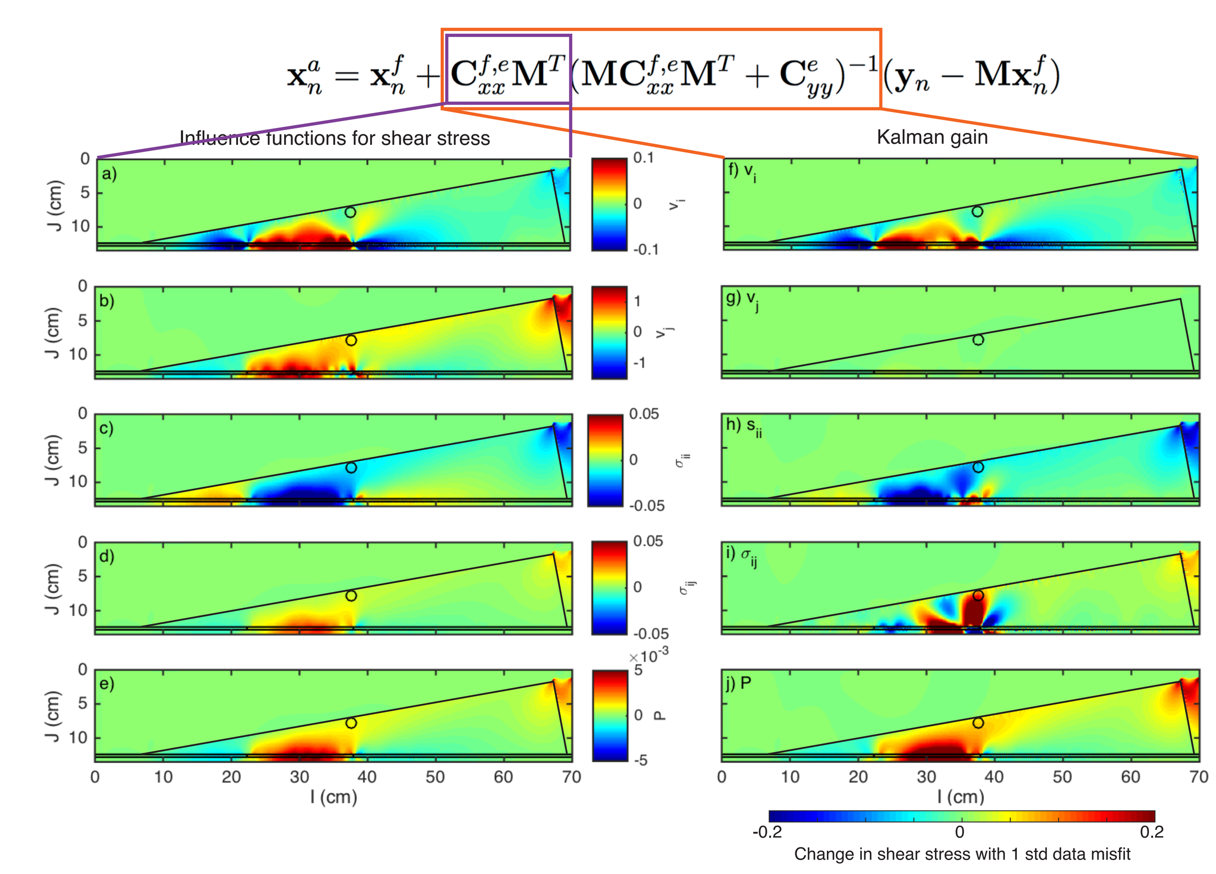

Fig. 9: **Information from physical model on how to update shear stress through space**
a-e) transposed influence functions show how shear stress covaries with observations of each type
f-j) scaled Kalman gain shows how to update shear stress due to a 1 std data misfit of each data type

Forecasting of these sythetic, analogue events works so well, because our prior knowledge of physical laws and observations are combined. Distinct added value is provided with respect to using observations or numerical models separately (Fig. 7).

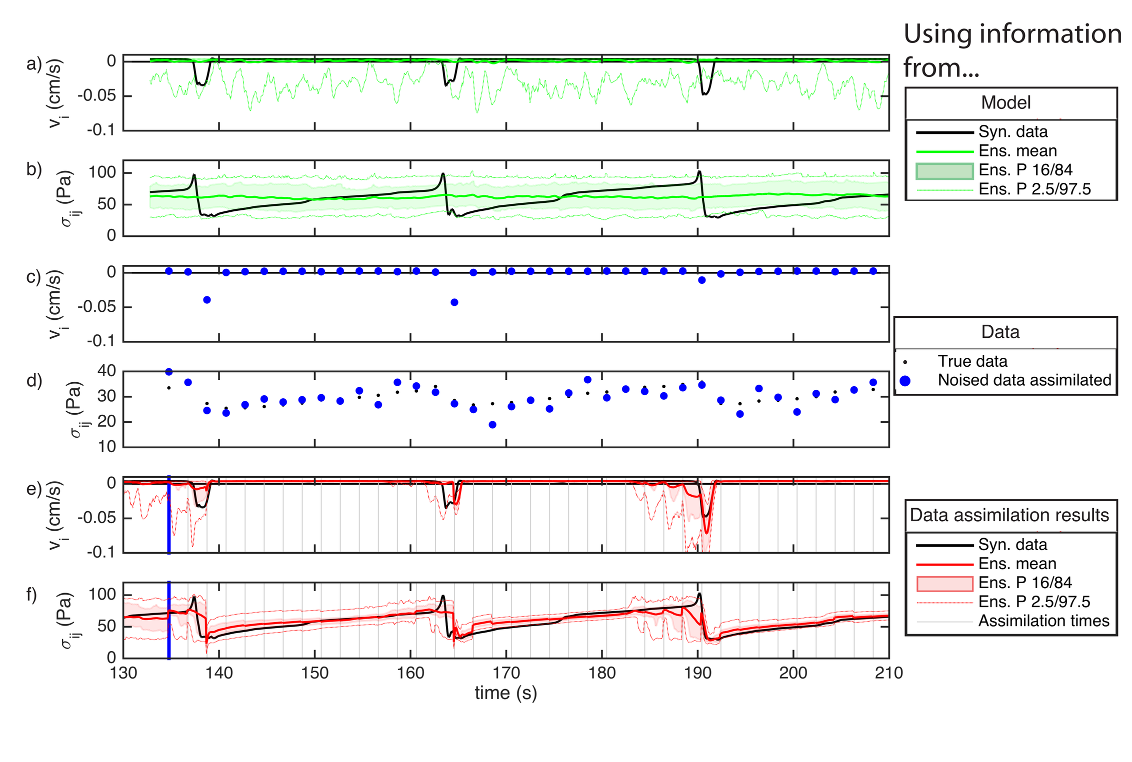

Fig. 7: **Zoom state evolution to distinguish contributions model and data.**
How many events do you estimate based on each panel?

References

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The corresponding manuscript will be submitted to GJI this week! It provides extensive explanations to make solid earth scientists understand sequential data assimilation.